MAJOR FACTS ABOUT
NORMAL SUBGROUPS AND FACTOR GROUPS

FACT 1. (Normal Subgroup Test) Let $G$ be a group and $H \leq G$ its subgroup. Then $H$ is normal in $G$ if and only if $xHx^{-1} \subset H$ for all $x \in G$.

FACT 2. (Factor Groups) Let $G$ be a group and $H \triangleleft G$ its normal subgroup. Then the set $G/H = \{aH | a \in G\}$ is a group under the operation $(aH)(bH) = abH$.

FACT 3. (Factor Group by the Center) Let $G$ be a group and $Z(G)$ its center. If $G/Z(G)$ is cyclic, then $G$ is Abelian.

FACT 4. Let $G$ be a group. Then $G/Z(G) \cong Inn(G)$.

FACT 5. (Cauchy’s Theorem for Abelian Groups) Let $G$ be a finite Abelian group and let $p$ be prime such that $p \mid |G|$. Then $G$ has an element of order $p$. 