Each group should write their solutions up together and turn in a single paper on Wednesday the 23rd. (There is no class on Monday the 21st.)

1. State the replacement theorem.

2. State the dimension theorem.

3. Suppose that \( T : V \to V \) is linear and that \( S = \{ v_1, \ldots, v_n \} \) is a subset of \( V \) such that \( \{ T(v_1), \ldots, T(v_n) \} \) is linearly independent. Show that \( S \) is linearly independent.

4. Find an example of a linear map \( T : \mathbb{R}^2 \to \mathbb{R}^2 \) such that \( N(T) = R(T) \).

5. Suppose that \( \beta = \{ v_1, \ldots, v_n \} \) is a basis for \( V \) and that \( T : V \to V \) is linear. Prove that if \( T \) is one-to-one, then \( \{ T(v_1), \ldots, T(v_n) \} \) is a basis for \( V \).