Math 24 Spring 2006 Quiz 1
Review Guide

A. To look at specifically (besides the homework, lecture notes, and book sections in general):

1. The true-false questions at the end of each section 1.2–1.6.
2. The key examples: $F^n$, $P_n(F)$ and $P(F)$, $M_{n \times m}(F)$, $\mathcal{F}(S, F)$.
3. Short proofs (you may be asked to reproduce them).

B. Some items to know:

1. Definition of group and field, uniqueness of identities and inverses, cancellation
2. Definition of vector space, how to tell whether a given set and operations form a vector space
3. Immediate consequence of the vector space definition: cancellation, uniqueness of zero and inverses, multiplication by scalar or vector zero gives result zero, inverse commutes strongly with scalar multiplication
4. Definition of subspace, how to tell whether a given subset of a vector space is a subspace
5. Standard examples of subspaces of $M_{n \times n}$: diagonal and symmetric matrices
6. Result of union or intersection of subspaces
7. Definition of linear combination, span, generate
8. How to set up and solve a system of linear equations given a linear combination of vectors with some unknowns
9. Definition of linear dependence, linear independence, relationship to linear combinations and span
10. How to use linear equations to determine whether a set of vectors is linearly independent
11. What linear dependence and independence of one set $S_1 \subset S_2$ tells you about the other set (if anything)
12. Definition of basis; relationship of basis size to size of linearly independent sets and spanning sets, number of representations of a vector of $V$ as linear combinations of its basis vectors
13. The standard bases for the key examples in A2 above (except $\mathcal{F}(S, F)$)
14. How to obtain a basis from a spanning set; the replacement theorem and its corollary about basis size
15. Definition of dimension, dimension of subspace
C. Some items not to memorize:
   (1) The definition of characteristic for a field; that will not be on the exam.
   (2) The definitions of trace, + (sum of vector spaces), and ⊕: if you need them I will provide their definitions.
   (3) Proofs. Even the short ones – memorizing them is a worse use of your time than simply trying to understand them and memorizing the usual tricks (adding 0 in some form, for instance).
   (4) Lagrange interpolation: we skipped this entirely.

D. Some ways to practice, if you’re looking for more work:
   (1) Invent collections of vectors in the standard vector space examples (where the field is \( \mathbb{R} \) or \( \mathbb{C} \) and calculate whether they are linearly dependent or independent.
   (2) Look at problems in the book similar to homework exercises: if I gave you some of the parts of a problem, try other parts. In particular, 1.2 #10–12, 16, 18–21; 1.3 #10–13, 22; 1.4 #5, 12, 16; 1.5 #8a, 10, 12–15, 17; 1.6 #4, 8, 9, 13, 14, 16, 17, 21
   (3) Try to write short proofs in your own words.