3.2
3. Clearly $A = 0$ implies $A$’s rank is 0. For the converse, if $A \neq 0$ it must have at least one nonzero column and hence the columns must span a vector space of dimension at least 1.

4. (b) rank is 2, matrix is 
\[
\begin{pmatrix}
1 & 0 \\
0 & 1 \\
0 & 0
\end{pmatrix}
\]

6. (a) $T^{-1}$ can also be expressed as $T(f(x)) = -(f(x) + 2f'(x) + f''(x))$.
(b) $T(1) = 0$, so noninvertible.
(d) $T^{-1}(b_1 + b_2 x + b_3 x^2) = (b_3, \frac{1}{2}(b_1 - b_2), \frac{1}{2}(b_1 + b_2) - b_3)$.
(f) Since $\text{tr}(A)$ and $\text{tr}(A^t)$ are equal, $T$ is not onto and hence noninvertible.

8. Let $v_i$ be the columns of $A$ and $w_i = cv_i$ be the columns of $cA$. If $x = a_1v_1 + \ldots + a_nv_n$, then because $c \neq 0$, $x = \frac{1}{c}(a_1w_1 + \ldots + a_nw_n)$.

14. (a) Show $R(T + U) \subseteq R(T) + R(U)$:
Let $x \in R(T + U)$. Then there is a $y$ with $(T + U)(y) = x$; i.e., $T(y) + U(y) = x$ and so $x \in R(T) + R(U)$.
(b) Show the rank of the sum is bounded by the sum of the ranks:
Certainly if $\beta$ is a basis for $R(T)$ and $\gamma$ a basis for $R(U)$, the set $\{b + 0, 0 + g : b \in \beta, g \in \gamma\}$ spans $R(T) + R(U)$, and so the dimension of $R(T) + R(U)$ is bounded by $\text{rank}(T)$ plus $\text{rank}(U)$. Then since $R(T + U)$ is a subspace by part (a) the result follows.
(c) .......

3.3
2. (b) \[
\begin{pmatrix}
\frac{1}{2} \\
\frac{2}{3} \\
1
\end{pmatrix}
\]

3. (a) Other vectors besides $(5,0)$ would work (e.g., $(2,1)$).
(b) \[
\left\{ \left(\frac{2}{3}, \frac{1}{2}, \frac{1}{2}, 0 \right) + a \left(\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, 1 \right) : a \in \mathbb{R} \right\}
\]

4. (a) $A^{-1} = \begin{pmatrix}
-5 & 3 \\
2 & -1
\end{pmatrix}$, solution $\begin{pmatrix}
-11 \\
5
\end{pmatrix}$

6. $(11/2, -9/2, 0)$ could be replaced, e.g. by $(0,1,-11)$