LECTURE OUTLINE
Multivariable Functions

Professor Leibon
Math 15
Oct. 25, 2004
Goals

Text, figure... Functions of Space

\[ f(x, y) \]
\[ \frac{\partial f}{\partial x}(x, y) \]
\[ f(x, t) \]
Mountain Range, graph

\[ f(x, y) = \cos(xy)e^{-\frac{x^2-y^2}{10}} \]
Contour Plot, topo map, level curves

\[ f(x, y) = \cos(xy) e^{-\frac{x^2 - y^2}{10}} \]
Notation

\[ f(x, y) = \cos(xy) e^{\frac{-x^2-y^2}{10}} \]

\[ f : \mathbb{R}^2 \rightarrow \mathbb{R} \]

The graph

\[ \{(x, y, f(x, y)) : (x, y) \in \text{Domain}\} \]
Partial Derivatives

\( \frac{\partial f}{\partial x}(x, y) \) means take the derivative in \( x \) viewing \( y \) as constant.

**Ex:** Find \( \frac{\partial f}{\partial x}(x, y) \) where \( f(x, y) = \cos(xy)e^{-\frac{x^2-y^2}{10}} \).
Time

\[ f(x, t) = e^{-\frac{x^2}{4t}} \]
Time

\[ f(x, t) = e^{-\frac{x^2}{4t}} \sqrt{\frac{4\pi t}{4x^2}} \]

Ex: Confirm \[ \frac{\partial f}{\partial t} = \frac{\partial^2 f}{\partial x^2}. \]
Time

\[ f(x, t) = \int_{-\infty}^{\infty} e^{-\frac{(x-y)^2}{4t}} \frac{1}{\sqrt{4\pi t}} g(y) \, dy \]

Ex: Explore the fact that \( \frac{\partial f}{\partial t} = \frac{\partial^2 f}{\partial x^2} \),
and \( f(x, 0) = g(y) \).