LECTURE OUTLINE
Practice Exam

Professor Leibon

Math 15

Oct. 11, 2004
Recall

\[
\frac{d\hat{r}}{dt} = \dot{\theta} \hat{\theta}
\]

\[
\frac{d\hat{\theta}}{dt} = -\dot{\theta} \hat{r}
\]

\[
\frac{d\hat{r}}{dt} = \ddot{r} \hat{r} + r \dot{\hat{\theta}} \hat{\theta}
\]

\[
\frac{d^2 \hat{r}}{dt^2} = (\ddot{r} - r \dot{\theta}^2) \hat{r} + (2 \dot{r} \dot{\theta} + r \ddot{\theta}) \hat{\theta}
\]

\[
\int \frac{1}{1+x^2} dx = \arctan(x) + C
\]
(Ex. 54) The coordinates of a vector in three-dimensional space is \((a, b, c)\).

(a) Express the vector as a sum of scalar multiples of the standard basis vectors.
(b) Find the magnitude (norm) of this vector.
(c) Find a unit vector in the direction of the given vector.
Problem 5 (Random and Slightly Modified Book Example)

(Ex. 20) An object is moving around the circle $x^2 + y^2 = 4$ in the $x, y$ plane (where the unit of distance is measured in meters) in a clockwise direction at a constant speed of 2 meters per second. Assume its initial position is $2\hat{i}$.

(a) Find its position after 5 seconds.

(b) Find a vector representing its velocity when it is located at the point with position vector $\sqrt{2}\hat{i} + \sqrt{2}\hat{j}$. 
Problem 6 (Slightly Modified Class Example: Cycloid, Ellipse, Pendulum....)

Suppose we have an ellipse and know that

$$\frac{d}{dt} \vec{r}(t) = \frac{e^2 d \sin(t)}{(1 + e \cos(t))^2} \hat{r} + \frac{ed}{1 + e \cos(t)} \hat{\theta}$$

Express this vector in Cartesian Coordinates.
Problem 7 (Theory Based: dot product rules, conservation of energy, work def...)

Use the definition of dot product and the rules of one dimensional calculus to justify that

$$\frac{d}{dt} (\vec{v} \cdot \vec{w}) = (\frac{d}{dt} \vec{v}) \cdot \vec{w} + \vec{v} \cdot (\frac{d}{dt} \vec{w}).$$
The force of gravity exerted by a massive object of mass \( M \) located at the origin of our three-dimensional axes on a small object of mass \( m \) located a distance \( r \) from the origin has magnitude \( \frac{mMg}{r^2} \) and acts directly towards the origin. Consider the work done by this force on the small object as it moves from the point \((0, 4, 0)\) to \((2, 0, 0)\) in a straight line.

(a) Without computation, should this work be positive or negative? Explain.
(b) Compute the work.
(c) Suppose I told you that this force was a conservative. Find a second, easier way to compute this force with this information. Do the computation.