LECTURE OUTLINE

Perturbation

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Math 15

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Goal

The Magenta Parabola
Approximation of Force at Equilibrium
Resonance
Solutions to $\frac{d^2f}{dt^2} + \gamma \frac{df}{dt} + cf = 0$. 

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Find a two solutions to $\frac{d^2 f}{dt^2} + \gamma \frac{df}{dt} + \frac{\gamma^2}{4} f = 0$. This is called the *Critically Damped* case.

**The famous guessing method:** If at first you don’t succeed, then *multiply by* $t^k$ and try try again.
Many physical systems can be approximated by a harmonic oscillator. For example, suppose we have a one dimensional conservative force $F(x) = -\nabla U(x)$. A point $a$ is an equilibrium point if it stays put, in other words $F(a) = 0$. We are often interested in what happen when we perturb a equilibrium solution. By Taylor approximation, near $a$ we have

$$m \frac{d^2(x - a)}{dt^2} \approx F(a) + \frac{\partial F}{\partial x}(a)(x - a) \approx \frac{\partial F}{\partial x}(a)(x - a).$$
Frequency of Small Oscillations

Example (Leonard-Jones Model): What is the frequency of small oscillations about equilibrium (the van der Waals radius) of a "particle of mass \( m \)" in a potential

\[
U(r) = \frac{a}{r^{12}} - \frac{b}{r^{6}}
\]

for \( a \) and \( b \) positive constants.
Exercise 1: What is the frequency of small oscillations about equilibrium of a "particle of mass $m$" in a potential

$$U(r) = \frac{2}{r^2} + r^2.$$
Exercise: Molecular Bonding Approximation

Exercise 2: (The Morse curve approximation in the covalent case) Let

\[ U(r) = A \left( e^{-2\alpha(r-r_0)} - 2e^{-\alpha(r-r_0)} \right) \]

where \( A, \alpha, \) and \( r_0 \) are positive constants particular to the molecule. What is the frequency of small oscillations about equilibrium?
Resonance

Find a general solution to

\[ D^2 f + \gamma D f + cf = \alpha \sin(\omega t) \]

Suppose the system is initially at rest and the above driving force is applied. Describe the system’s behavior. What does the system look like for large time?