Each problem is worth 7.5 points.

1. Which of the following equations represents the plane passing through 
   \((-5, 2, 3)\) and orthogonal to the line \((2 + t, -7t, -11 + 2t)\)?
   (a) \(-5(x - 1) + 2(y + 7) + 3(z - 2) = 0\)
   (b) \(x - 7y + 2z = -13\)
   (c) \(x - 7y + 2z = -3\)
   (d) \(x + y + z = 1\)

2. An equation of the tangent line to the curve \(c(t) = (t^2 - 1, \cos(t^2), t^4)\) at the point \((\pi - 1, -1, \pi^2)\) is:
   (a) \((2\sqrt{\pi}t - \pi - 1, -2t, 4\pi\sqrt{\pi}t - 3\pi^2)\)
   (b) \((2\sqrt{\pi}t + \pi - 1, -2t, 4\pi\sqrt{\pi}t + \pi^2)\)
   (c) \((2\sqrt{\pi}t - \pi - 1, -1, 4\pi\sqrt{\pi}t - 3\pi^2)\)
   (d) \((2\sqrt{\pi}t + \pi, -1, 4\pi\sqrt{\pi}t + \pi^2)\)

3. Match the following functions with their level curves \(f(x, y) = k, \ k = 1, 2, 3, 4, \ldots\)
   (1) \(f(x, y) = (x^2 + y^2)^{1/2}\) (i) unequally spaced concentric circles
   (2) \(f(x, y) = 4 - 3x + 2y\) (ii) unequally spaced lines
   (3) \(f(x, y) = 2x^2 + 2y^2\) (iii) concentric ellipses
   (4) \(f(x, y) = x^2 + 2y^2 + 1\) (iv) equally spaced concentric circles

   (a) 1-(iv), 2-(ii), 3-(i), 4-(iii)
   (b) 1-(i), 2-(ii), 3-(iv), 4-(iii)
   (c) 1-(iii), 2-(ii), 3-(i), 4-(i)
   (d) 1-(ii), 2-(ii), 3-(iv), 4-(iii)

4. An equation of the tangent plane to the surface \(x^2 + y^2 + xy\sin z - 3 = 0\) at the point \((1, -2, \frac{\pi}{2})\) is:
   (a) \(y + z = 1 + \frac{\pi}{12}\)
   (b) \(3y + 2z = 6 + \frac{\pi}{2}\)
   (c) \(y = -2\)
   (d) \(3y = 2\)

5. Consider the functions \(f(u, v) = e^{uv}\) and \(g(x, y) = (x + y, x - y)\). The derivative matrix of \(f \circ g\) at \((x, y) = (1, 1)\) is equal to:
   (a) \[
   \begin{bmatrix}
   1 & 1 \\
   1 & -1 \\
   \end{bmatrix}
   \]
   (b) \[
   \begin{bmatrix}
   0 & 1 \\
   \end{bmatrix}
   \]
   (c) \[
   \begin{bmatrix}
   0 & 1 \\
   0 & 1 \\
   \end{bmatrix}
   \]
   (d) \[
   \begin{bmatrix}
   2 & -2 \\
   \end{bmatrix}
   \]
6. The directional derivative of \( f(x, y, z) = \frac{1}{x^2+y^2+z^2} \) at the point \((2, 3, 1)\) in the direction of \( \mathbf{v} = 2 \mathbf{i} + \mathbf{j} - 2 \mathbf{k} \) is:
   (a) \(-\frac{5}{211\pi}\)
   (b) \(\frac{5}{21\pi}\)
   (c) \(-\frac{5}{7\pi}\)
   (d) \(\frac{5}{7\pi}\)

7. The length of the curve \( c(t) = t \mathbf{i} + \ln t \mathbf{j} + 2\sqrt{2t} \mathbf{k} \) for \( 1 \leq t \leq 2 \) is:
   (a) \(2 + \ln 2\)
   (b) \(\frac{19}{2} + \frac{\ln 3.8}{2}\)
   (c) \(1 + \ln 2\)
   (d) \(2\)

8. Which of the following implies the vector field \( F = F_1 \mathbf{i} + F_2 \mathbf{j} + F_3 \mathbf{k} \) is not a gradient?
   (a) \(\frac{\partial F_1}{\partial y} = \frac{\partial F_2}{\partial x}, \frac{\partial F_3}{\partial z} = \frac{\partial F_2}{\partial x} \) and \(\frac{\partial F_3}{\partial y} = \frac{\partial F_1}{\partial y}\)
   (b) \(\nabla \times F = 0\)
   (c) \(\int_{C_1} F \cdot ds = \int_{C_2} F \cdot ds\), where \(C_1\) and \(C_2\) are any two curves with common start point and common end point.
   (d) \(\int_{C} F \cdot ds = 1\), where \(C\) is a closed curve.

9. The volume of the solid below the surface \( z = (xy)^2 \cos(x^3) \) and above the region \( R \) in the \( xy \)-plane given by \( R = [0, \sqrt{\frac{3}{4}}] \times [0, 1] \) is equal to:
   (a) \(\frac{\sqrt{2}}{18}\)
   (b) \(\frac{1}{18}\)
   (c) \(\frac{\sqrt{2}}{6}\)
   (d) \(\frac{\sqrt{2}}{12}\)

10. The value of the integral \( \int_{0}^{\frac{\pi}{2}} \int_{0}^{2} e^{x^2} y \, dy \, dx \) is:
    (a) \(-\frac{2}{3}(e^8 + 1)\)
    (b) \(\frac{2}{3}(e^8 - 1)\)
    (c) \(\frac{2}{3}e^{64}\)
    (d) \(\frac{2}{3}e^{64} - \frac{2}{3}\)

11. The value of the integral \( \int \int_{D} \cos(x^2 + y^2) \, dx \, dy \), where \( D \) is the region defined by \( x^2 + y^2 \leq 1 \), is:
    (a) cannot be evaluated
    (b) \(\pi \sin 1\)
    (c) \(2\pi \sin 1\)
    (d) \(\pi\)

12. The value of the integral \( \int \int_{W} \sqrt{x^2 + y^2 + z^2} \, dx \, dy \, dz \), where \( W \) is the solid bounded by \( x^2 + y^2 + z^2 = 4 \) and \( x^2 + y^2 + z^2 = 9 \), is:
    (a) \(65\pi\)
    (b) \(5\pi^2\)
    (c) \(\frac{20}{3}\pi\)
    (d) \(10\pi\)
13. The work done by the force field \( F(x, y, z) = x \mathbf{i} + y \mathbf{j} \) when a particle is moved along the path \((3t^2, t, 1)\) from \((12, 2, 1)\) to \((3, 1, 1)\) is equal to:
   (a) 69
   (b) 70
   (c) -70
   (d) -69

14. Consider the surface given by \((3\cos \theta \sin \phi, 2\sin \theta \sin \phi, \cos \phi), \ \theta \in [0, 2\pi]\) and \(\phi \in [0, \pi]\). An equation for the tangent plane to this surface at the point \((\frac{3}{\sqrt{2}}, 0, \frac{1}{\sqrt{2}})\) is given by:
   (a) \(x + y = -\frac{4}{\sqrt{2}}\)
   (b) \(x + \frac{3}{2}y + z = 3\sqrt{2}\)
   (c) \(x + 2z = 3\sqrt{2}\)
   (d) \(y = 0\)

15. The area of the cone \(z^2 = x^2 + y^2\) lying in the region of space defined by \(x \geq 0, y \geq 0\) and \(0 \leq z \leq 1\) is:
   (a) \(\frac{\sqrt{2} \pi}{4}\)
   (b) \(\frac{\sqrt{2} \pi}{2}\)
   (c) \(\frac{\pi}{4}\)
   (d) \(\sqrt{2}\)

16. The surface integral \(\iint_S F \cdot d\mathbf{S}\), where \(F = \mathbf{i} + \mathbf{j} + z(x^2 + y^2)^2 \mathbf{k}\) and \(S\) is the surface \(x^2 + y^2 = 1, 0 \leq z \leq 1\), is equal to:
   (a) \(\pi\)
   (b) \(\frac{14}{25}\)
   (c) \(0\)
   (d) \(2\pi\)

17. The value of \(\int_C (2x^3 - y^3)dx + (x^3 + y^3)dy\), where \(C\) is the positively oriented unit circle centered at the origin, is:
   (a) \(\frac{3\pi}{2}\)
   (b) \(0\)
   (c) \(\frac{1}{2}\)
   (d) \(\pi\)

18. Let \(D\) be a region in the plane and \(\partial D\) the positively oriented boundary of \(D\). Which of the following expressions does not represent the area of \(D\)?
   (a) \(\int_{\partial D} (y^2 - 1)dx + (xy^2)dy\)
   (b) \(\int_{\partial D} \left(\frac{y^2}{3} - y\right)dx + (xy^2)dy\)
   (c) \(\int_{\partial D} x \ dx + (x + y)dy\)
   (d) \(\int_{\partial D} x \ dy\)

19. The value of \(\iint_S (\nabla \times F) \cdot d\mathbf{S}\), where \(S\) is \(x^2 + y^2 + z^2 = 9, z \geq 0\) with normal in the positive \(z\) direction and \(F = x\mathbf{i}\), is:
   (a) 0
   (b) \(9\pi + \frac{9}{2}\)
   (c) \(9\pi\)
   (d) \(18\pi\)
20. The flux of \( F = 3xy^2i + 3x^2yj + z^3k \) out of the sphere \( x^2 + y^2 + z^2 = 1 \) is equal to:

(a) \( \frac{6\pi}{5} \)
(b) \( \frac{12\pi}{5} \)
(c) \( \frac{4\pi}{3} \)
(d) 0