1. For a group $G$, $\text{Tor}(G) = \{g \in G \mid g^n = e \text{ for some } n \geq 1\}$ is called the set of torsion elements of $G$. Of course this really is only interesting for infinite groups.

   (a) If $G$ is abelian, show that $\text{Tor}(G)$ is a subgroup of $G$, called its torsion subgroup.

   (b) If $G$ is not abelian, show that $\text{Tor}(G)$ need not be a subgroup of $G$. One can find a nice counterexample in $G = SL_2(\mathbb{Z}) = \langle S, T \rangle$ where $S = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$, and $T = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$. Hint: $ST$ is a nice element.

2. For $n \geq 3$, characterize the center of the symmetric group $S_n$.

3. For $n \geq 5$, show that the only normal subgroups of $S_n$ are $\{e\}$, $A_n$, and $S_n$. Use this to show that $S_n$ is not solvable for $n \geq 5$. This fact is key in showing that the general polynomial of degree $n \geq 5$ is not solvable by radicals.

4. Let $p < q$ be primes and $G$ a nonabelian group of order $pq$. Show that there is an embedding of $G$ into the symmetric group $S_q$. 