1. For $n \geq 3$, characterize the center of the symmetric group $S_n$.

2. For $n \geq 5$, show that the only normal subgroups of $S_n$ are $\{e\}$, $A_n$, and $S_n$. Use this to provide a proof different than Lang’s that $S_n$ is not solvable for $n \geq 5$.

3. For $p$ and $q$ distinct primes, show that any group of order $p^2q$ is solvable.

4. Let $G$ be a group of order 12, and assume that $G$ has more than one Sylow 3-subgroup. Show that $G \cong A_4$. Hint: Letting $G$ act on the set of Sylow 3-groups with provide a homomorphism $\varphi : G \to S_4$. 
