An Evaluation Report on the
Workshop in Teaching Introductory Combinatorics by Guided Discovery
Dartmouth College
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In August 2003 seventeen mathematicians spent a week at Dartmouth College to learn how to teach mathematics, especially combinatorics, by the guided discovery method. Guided discovery conjoins two strategies—self-paced learning and group work—in an effort to make learning more efficient for all students. The carefully structured problem sequence in guided discovery is designed to carry students step by manageable step through the material, so that they construct the mathematical concepts in a cyclical "bottom up" approach, instead of receiving it in the "top down" lecture method (and then hoping to deconstruct and understand by solving problems). The workshop was led by Ken Bogart, who piloted the method in combinatorics classes at Dartmouth College and wrote the accompanying text under the auspices of a National Science Foundation grant, and Karen Collins, a member of the project's advisory board who had used Bogart's method at Wesleyan University.

The week's activities centered around conversations with those who had experience in guided discovery, either as instructor (Bogart, Collins, Rosa Orellana), student (Lisa Birzen) or evaluator (Jane Korey) and practice sessions in which participants worked on combinatorics problems as students would in guided discovery groups. Selected readings presented the theoretical basis for guided discovery and grounded group discussions about the method. There was a special session on providing students with feedback and an opportunity to practice grant writing for those who might seek support for this activity at their own campus. On the last day, participants were asked to comment on various goals of the workshop and their responses indicate that the workshop was highly successful in communicating the method and in motivating faculty to undertake teaching in this way, both with combinatorics and with other mathematics. In addition, participants had fun. They had only praise for the organization and amenities of the workshop, and for the resulting hard-working camaraderie among their colleagues in learning.

**Why they came.** The goals of most participants were closely aligned with the workshop's focus: more than half (10) came specifically because they wanted to teach

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combinatorics by guided discovery using Bogart's book. Seven of these ten, and one other participant, also wanted to learn how to teach other mathematics by this method; six were interested in learning how to create materials to use in guided discovery. But five who enrolled were more tentative in their interest; they wanted to "explore the possibility," "find out more," or get "an introduction" to the method. (One person wanted to learn how to adapt the method to high school students.) By the end of the workshop, all of these five were strongly committed to using guided discovery in their classrooms.

Understanding the method  By their own assessments, participants came away with a solid understanding of the pedagogical rationale for guided discovery and what teaching by the method entails. We asked them to agree or disagree, on a five point scale, with the following statement, "After this workshop, I understand what teaching by guided discovery entails and the pedagogical rationale for doing so." Ranking their understanding on a scale where "5" indicated the greatest and "1" the least understanding, the mean score of 4.4 indicates substantial accomplishment. Expectably, questions remained and most of the questions concerned group work, an aspect of the method that is less familiar to faculty and less well understood in general. Participants worried particularly about balancing group work and lecture in a productive way, but some also felt somewhat uncertain about how to form and manage groups, how to deal with group dysfunction, and when to offer assistance and when to withhold. One remained unclear about the theoretical grounding for guided discovery's claims; another wondered how the instructor's personality influenced the method. Several remained unsure about how to create materials and sequence problems. Two pointed out that lacunae in their knowledge and skills would most likely be revealed in practice. As one wrote, "I think most aspects are clear theoretically, but perhaps will not be when I actually need to implement them."

Developing skills. We asked participants to agree or disagree with the statement "After this workshop I have the skills I need to teach mathematics by the guided discovery method," on a five point scale where "5" indicated strong skills development and "1" indicated weak. While the mean score of 3.9 suggests somewhat less confidence in their skills than in their theoretical grasp of the method, it is nonetheless a strong outcome for a week's introduction. Most participants felt secure in their ability to structure and manage groups; eight cited this as a skill they felt "most confident" about. Seven mentioned being able to guide groups skillfully through the material, a matter of intellectual rather than social organization. Others mentioned grading (2), using the materials (2), and creating interesting problems (1). They were least confident about those aspects of the method that asked them to give up some control over what happens in the classroom. Three were not sure how to deal with students who were uncooperative in group work. Three others worried about being able to provide quick insights into novel or unanticipated mathematical questions. As one wrote, "I'm not quite sure how to react if it doesn't work out in class. As the authority figure in the class, what would students think if they see you 'fumbling a bit'?" But one saw the challenge not as one of preserving authority, but of relinquishing it. "[I feel least confident about] changing my personal style to encourage students to accept some of the responsibility and authority of mathematics." Five still felt a little shaky
about the mechanics of group work: how much help to provide, when to step back and summarize—and how to do that—and how to manage time. None of these is an issue in a lecture format, where the instructor is in complete charge of the material and the schedule, and the students' good behavior (if not their full attention) is presumed. Two participants did not feel confident about creating new materials.

**How they learned.** Overwhelmingly, participants said their understanding of the method and their skill development emerged from group work. Sixteen of the seventeen cited working on problem sequences in a group, just as students would do, as the activity that contributed most to their learning. As one succinctly wrote, "The actual group work exhibited the method best to me." Another said, "[I learned most from] working in groups myself. I think I'll be able to observe group interaction better, and also to realize some of the distractions that can occur." But seven also noted that Bogart's and Collins' reflections about their experiences with the method were very helpful. Three cited the opportunity to practice teaching by guided discovery as particularly useful, and four mentioned insights into the students' perspective provided by the videos of Bogart's class and the conversation with a guided discovery alumna.

Almost all felt they would have benefited if they had received the readings before the workshop, so that they arrived with a foundation to build on. When asked what would have helped them learn better, they said, in essence, "more;" more group work, more carefully constructed working groups (matching skills and backgrounds)—and opportunities to work with more than one group, more time to create and test materials and more opportunities to practice teaching by guided discovery. One pointed out that the next step in the learning process was simply "to do it myself ands see what happens!"

**Creating materials.** Learning how to develop one's own problem sequences for guided discovery was an optional activity on the last morning of the workshop, and only a dozen participants took advantage of this. Those who did received a useful introduction to the process, but expectably, after two hours of practice, few felt ready to create materials on their own. Asked to rate how capable they felt about developing materials for guided discovery, where "5" represented "very capable" and "1" represented "not capable," the mean rating was 3.4. Participants said that Bogart's discussion of his experience in developing materials provided the most insight into the process; also useful was his book, which offered a finished example of it. Several found discussions with other participants helpful; two cited the readings. Faculty said they simply needed to spend more time developing problems—and getting peer feedback on them—to get a firmer grip on the process.

**Ready to teach?** Finally, we asked participants to agree or disagree with the statement "After this workshop I feel ready to teach by guided discovery." The mean response was 4.1 on a five point scale where "5" indicated great readiness and "1" indicated unreadiness. Clearly, these mathematicians felt confident about introducing the guided discovery method into their classrooms. And many endorsed having another workshop to which they could bring new insights and questions that emerged from their first guided discovery efforts, do more practice teaching, and spend more time learning how to develop materials, especially for other mathematics courses.
Conclusions. In a week's time, Bogart and Collins' guided discovery workshop prepared participants to employ this student-centered approach in their own classrooms. Whether they arrived already intending to teach combinatorics by guided discovery using Bogart's book or came with a less focused curiosity about the method, all left with the knowledge and skills—and the confidence—to use the method on their own. The workshop format emphasizing hands-on guided discovery experience in the role of the student seemed notably successful in communicating both the teaching method and the nature of the student experience. Bogart's and Collins' conversations about their own experiences were also highly useful in conveying the new practices the method entails and the novel situations likely to arise. Participants left with a working command of strategies for forming and managing groups, and for guiding students through a problem sequence, with hints, feedback and summaries as needed. The short introduction to creating materials for a guided discovery course whetted the appetite of many. The desire to spend more time creating problem sequences, often for courses other than combinatorics, led the list of topics to be considered in a follow-on workshop.