Math 354 Summer 2004

Midterm #1

Monday, 7/26/2004

Name: _____

Problem	Points
1	
2	
3	
4	
5	

Homework grade	
Quiz grade	
Midterm #1 grade	
Projected grade	

Math 354 Summer 2004 **Midterm #1**

1 Consider the following scenario. Model it as a linear programming problem. Be sure to state explicitly what each of your decision variables $x_1, x_2, \ldots$ represent. Do **not** attempt to solve the problem.

Marion Jones, who won three gold medals in the 2000 olympics, was recently accused by her ex-husband C.J. Hunter of using a mixture of four banned performance-enhancing drugs (often referred to as "stacking") during those olympics: human growth hormone (HGH), anabolic steroids (AS), the endurance booster erythropoietin (EPO), and insulin. Jones had previously been suspended for four years after failing to show up for a random drug test, but her suspension was dropped after an appeal in which she was represented by Johnnie Cochran. Hunter himself tested positive for steroids on four separate occasions in 2000.

Let's say that Jones can use a mixture of three designer drugs, each of which contains a blend of the four substances:

	HGH	AS	EPO	insulin
Type A	10	5	20	5
Type B	3	20	10	0
Type C	20	5	5	5

She would like maximize the total number of units of the four substances she uses, but she doesn't want to get caught or die. So she needs to pass the tests for HGH, AS, and EPO, which require that she ingest at most 60, 40, and 100 units of the three drugs, respectively. Since insulin passes through the body extremely quickly, it is nearly impossible to test for, so Marion doesn't need to worry about that. However, an overdose of insulin causes the body to break down glucose far too quickly, resulting in hypoglycemia, the symptoms of which range from nausea to death. For this reason, Marion should limit her insulin intake to at most 35 units.

2 Convert the following linear programming problem into **both** standard form and canonical form. Do **not** attempt to solve the problem.

Minimize $z = x_1 - 3x_2 + 7x_3$
subject to

$$\begin{array}{rcrcrcl} x_1 & - & 2x_2 & + & 4x_3 & \leq & 916 \\ 10x_1 & & & + & 15x_3 & \geq & 1972 \\ x_1 & + & x_2 & + & 6x_3 & = & 123 \\ x_1, & & & & x_3 & \geq & 0 \\ & & x_2 & & & & \text{unconstrained} \end{array}$$

3 Solve the following linear programming problem by using the **Extreme Point Theorem**. That is, graph the set of feasible solutions, find the extreme points, and find the optimal solution(s).

$$\begin{array}{l}\text{Maximize } z = 2x + 3y \\ \text{subject to} \\ \begin{array}{rcrcl} x & + & y & \leq & 3 \\ -x & + & y & \leq & 2 \\ x, & & y, & \geq & 0 \end{array}\end{array}$$

4 Solve the following linear programming problem by the **simplex method**. No points will be given for any other method!

$$\begin{array}{l}\text{Maximize } z = 3x_1 + 2x_2 + 4x_3 \\ \text{subject to} \\ \begin{array}{rcrcrcl} x_1 & + & x_2 & + & 2x_3 & \leq & 4 \\ 2x_1 & & & + & 3x_3 & \leq & 5 \\ 2x_1 & + & x_2 & + & 3x_3 & \leq & 7 \\ x_1, & & x_2, & & x_3 & \geq & 0 \end{array}\end{array}$$

Answer:

	x_1	x_2	x_3	u_1	u_2	u_3	
u_1	1	1	2	1	0	0	4
u_2	2	0	3	0	1	0	5
u_3	2	1	3	0	0	1	7
	-3	-2	-4	0	0	0	0

	x_1	x_2	x_3	u_1	u_2	u_3	
u_1	$-\frac{1}{3}$	1	0	1	$-\frac{2}{3}$	0	$\frac{2}{3}$
x_3	$\frac{2}{3}$	0	1	0	$\frac{1}{3}$	0	$\frac{5}{3}$
u_3	0	1	0	0	-1	1	2
	$-\frac{1}{3}$	-2	0	0	$\frac{4}{3}$	0	$\frac{20}{3}$

	x_1	x_2	x_3	u_1	u_2	u_3	
x_2	$-\frac{1}{3}$	1	0	1	$-\frac{2}{3}$	0	$\frac{2}{3}$
x_3	$\frac{2}{3}$	0	1	0	$\frac{1}{3}$	0	$\frac{5}{3}$
u_3	$\frac{1}{3}$	0	0	-1	$-\frac{1}{3}$	1	$\frac{4}{3}$
	-1	0	0	2	0	0	8

	x_1	x_2	x_3	u_1	u_2	u_3	
x_2	0	1	$\frac{1}{2}$	1	$-\frac{1}{2}$	0	$\frac{3}{2}$
x_1	1	0	$\frac{3}{2}$	0	$\frac{1}{2}$	0	$\frac{5}{2}$
u_3	0	0	$-\frac{1}{2}$	-1	$-\frac{1}{2}$	1	$\frac{1}{2}$
	0	0	$\frac{3}{2}$	2	$\frac{1}{2}$	0	$\frac{21}{2}$

5 Solve the following linear programming problem by the **two-phase simplex method**. No points will be given for any other method!

$$
\begin{array}{l}
\text{Maximize } z = 3x_1 + x_2 \\
\text{subject to} \\
\begin{array}{rcrcl}
x_1 & - & x_2 & \leq & -1 \\
-x_1 & - & x_2 & \leq & -3 \\
2x_1 & + & x_2 & \leq & 4 \\
x_1, & & x_2, & \geq & 0
\end{array}
\end{array}
$$