231-avoiding permutations and the Schensted correspondence

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The Schensted correspondence gives a bijection between a permutation $\sigma \in S_n$ and a pair of Young tableaux of the same shape λ, which is a partition of n. Given a permutation σ, the corresponding pair of tableaux may be computed by the well-known Robinson-Schensted-Knuth algorithm. I will present generating functions for 231-avoiding permutations that contain information about the entire shape of the Schensted correspondence. In particular, this provides information about the longest unions of any number of disjoint increasing or disjoint decreasing subsequences. Generating functions for 231-avoiding permutations by peaks, valleys, inversions, and longest increasing subsequence will also be presented.

For $\sigma \in S_n$, denote by $\lambda_i(\sigma)$ the number of cells in the i-th row of the Schensted correspondence for σ. Define the *weight* of σ, denoted $w(\sigma)$, by $w(\sigma) = \prod_i x_i^{\lambda_i(\sigma)}$. Let $\mathrm{inv}(\sigma)$ denote the number of inversions in σ. Then we obtain the following result.

Theorem 1. *Let $T_{k,n}$ be the set of 231-avoiding permutations $\sigma \in S_n$ whose Schensted correspondence has at most k rows, and let $\mathbf{x} = (x_1, x_2, \ldots)$. Then let*

$$f_k(\mathbf{x}, q, z) = \sum_{n \geq 0} \sum_{\sigma \in T_{k,n}} w(\sigma) q^{\mathrm{inv}(\sigma)} z^n.$$

f_k satisfies the following recurrence.

$$f_k(z) = \left(1 - x_1 z - \sum_{j=2}^{k} x_j z (f_{j-1}(\mathbf{x}, q, zq) - f_{j-2}(\mathbf{x}, q, zq))\right)^{-1}$$

$$f_0(z) = 1$$

In particular, we note that f_k is a rational function for all k.