Permutation patterns and the Möbius function

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A permutation is separable if it can be generated from the permutation 1 by successive sums and skew sums or, equivalently, if it avoids the patterns 2413 and 3142. Using the decomposition into sums and skew sums, we give a formula for the Möbius function of an interval (q, p) in the poset of separable permutations ordered by pattern containment. This formula computes the Möbius function for such intervals in polynomial time.

A consequence of the formula is that the Möbius function of such an interval (q, p) is bounded by the number of occurrences of q as a pattern in p. The formula also implies that for any separable permutation p the Möbius function of (1, p) is either 0, 1 or -1, in addition to allowing straightforward computation of the Möbius function for many interesting special cases. The formula of Sagan and Vatter for the Möbius function of layered permutations is a special case of our formula.

We also present some results for the Möbius function of non-separable permutations.

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