Consecutive patterns: From permutations to column-convex polyominoes and back

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We expose the ties between the consecutive pattern enumeration problems associated with permutations, compositions, column-convex polyominoes (CCPs), and words. More precisely, if $\mathcal{PS}$, $\mathcal{PC}$, $\mathcal{PCCP}$, and $\mathcal{PW}$ respectively denote the sets of consecutive pattern enumeration problems on permutations, on compositions, on column-convex polyominoes, and on words, then

$$\mathcal{PS} \subset \mathcal{PC} \subset \mathcal{PCCP} \subset \mathcal{PW}. \tag{7}$$

The significance of (1) is that it allows powerful methods from the larger problem sets to be applied to the smaller problem sets. To illustrate, we will show how various results on words as well as Bousquet-Mélou's adaptation of Temperley's method for enumerating CCPs may be used to count permutations by consecutive patterns.

In particular, we exploit the perspective of (1) to q-count permutations by (i,d)-peaks, up-down type, uniform m-peak ranges, and (i,m)-maxima. Notably, our approach provides a solution to the $(2m+1)$-alternating pattern problem on permutations posed by Kitaev. We will also show that the generating function for permutations by a given pattern is deducible from the generating function for a related pattern permutation set; for instance, the generating function for permutations by peaks may be obtained from the one for up-down permutations of odd length.

Besides providing an expose of (1), we also initiate the explicit study of CCPs by consecutive (or ridge) patterns. Our introduction of two-column ridge patterns provides a unifying characterization of the common subclasses of CCPs. Examples of consecutive pattern distributions on directed CCPs and on CCPs will be presented.

This is joint work with Mark Tiefenbruck (University of California).