The simple permutation poset

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Simple permutations can be naturally represented as a poset where the order relation is the usual pattern containment relation on permutations. In this article, we show that the poset restricted to non exceptional permutations is graded. Moreover, if σ and π are two simple non-exceptional permutations such that $\pi \prec \sigma$ then there exist simple permutations $\tau_0 = \sigma, \tau_1, \ldots, \tau_k = \pi$ such that $\tau_i \prec \tau_{i-1}$ and $|\tau_i| = |\tau_{i-1}| - 1$. The following figure represents the poset of simple permutations σ such that $\sigma \prec 27481635$.



A detailed study of the links between two consecutive levels of the poset allow to compute the probability of remaining simple when deleting a random element in a simple permutation. Indeed, we are able to prove that on average n - c elements can be removed in a simple permutations and the obtained permutation is still simple.

This is joint work with Dominique Rossin.