# An asymptotic version of a theorem of Knuth

*Jonathan Novak* University of Waterloo

One of the earliest results in the theory of pattern avoidance is the enumeration of permutations with no decreasing subsequence of length three: they are are counted by the Catalan numbers. The first published proof of this fact is due to Knuth in the 1960's. A later and a priori much more difficult result is Regev's 1981 asymptotic estimate of the number of permutations of $[N]$ which have no decreasing subsequence of length $d+1$, in the large $N$ limit with $d$ fixed. I will discuss the following recent result which provides a conceptual link between these formulas of Knuth and Regev: for any fixed positive integer $d$, the number of permutations in the symmetric group $S(dn)$ with no decreasing subsequence of length $d+1$ is asymptotically equal, for large $n$, to the number of standard Young tableaux on the $d$ by $2n$ rectangle. This asymptotic equivalence allows one to obtain Regev's estimate by a direct application of Stirling's formula. The proof of the stated asymptotic equivalence amounts to a physically obvious symmetry in the energy functional of a Coulomb gas on the line confined by a potential well at zero. If time permits we will discuss some connections between the "rectangle" estimate and the Tracy-Widom law, which emerges in the asymptotics of permutations without long decreasing subsequence when one takes a double scaling limit.