Composition and partition matrices, bivincular patterns and (2+2)-free posets

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In this talk we will present an overview of results from three related papers, with an emphasis on the paper that introduces composition and partition matrices. These two new types of matrices are matrix analogues for set partitions. A composition matrix is an upper triangular matrix in which the entries partition the set $\{1, ..., n\}$, and for which there are no rows or columns containing only empty sets. A partition matrix is a composition matrix in which an order is placed on where entries may appear relative to one-another.

We show that partition matrices are in one-to-one correspondence with inversion tables. Non-decreasing inversion tables are shown to correspond to partition matrices with a row ordering relation. Partition matrices which are s-diagonal are classified in terms of inversion tables. The 2-diagonal partition matrices are enumerated using the transfer-matrix method and are equinumerous with permutations which are sortable by two pop-stacks in parallel.

We show that composition matrices are in one-to-one correspondence with labeled $(2+2)$-free posets. Labeled $(2+2)$-free posets which are generated from non-decreasing ascent sequences are shown to be in one-to-one correspondence with parking functions. Finally, we show that the set product of ascent sequences and permutations are in one-to-one correspondence with $(2+2)$-free posets in which elements are the cycles of a permutation, and use this relation to give an expression for the number of labeled $(2+2)$-free posets.