

A POLYNOMIAL ALGORITHM FOR DECIDING THE FINITENESS OF THE NUMBER OF SIMPLE PERMUTATIONS IN PERMUTATION CLASSES

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Our goal is to provide automatic methods to enumerate permutation classes. More precisely, in [1], the authors explain how to compute the generating function of a permutation class whenever this class contains a finite number of simple permutations. In [5, 4, 6], the authors prove that determining if a class given by its basis contains a finite number of simple permutations is decidable. The proof relies on a subclass of permutations called pin-permutations. In [3], we characterize the pin-permutations in terms of decomposition trees and give their generating function. In [2], we use this characterization to give a $\mathcal{O}(n \ln n)$ algorithm to determine if a wreath-closed class of permutations given by its basis contains a finite number of simple permutations. This result was obtained by proving that determining if a simple permutation contains a given simple pin-permutation can be reduced to a problem on words, namely determining if a word is a factor of another one.

In this article, we prove a polynomial algorithm to determine if a general permutation class contains a finite number of simple permutations. In [6], the authors reduce the problem of containing a finite number of simple permutation to a co-finiteness problem on regular languages. These languages are given by nondeterministic finite automata leading to an exponential complexity in the decision procedure. We use the same encoding of permutations by words. Our characterization of pin-permutations obtained in [3] allows us to provide an explicit description of the set of words associated to any pin-permutation. This description is essential in the efficient construction of complete deterministic automata -instead of nondeterministic ones- for which the time complexity of the co-finiteness problem is linear.

Putting all together we can prove the following theorem and also provide an effective algorithm:

Theorem 1. *Deciding if a permutation class $\mathcal{C} = Av(\sigma^{(1)}, \sigma^{(2)}, \dots, \sigma^{(k)})$ contains a finite number of simple permutations can be done in time $\mathcal{O}(n^{3k})$ where n is the size of the largest permutation in the basis.*

This is joint work with Frédérique Bassino, Adeline Pierrot and Dominique Rossin.

References

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