Math 8
Winter 2015
Written Homework Guidelines

The written homework for Math 8 is assigned daily, generally one problem per day, and due weekly, as explained on the course web page.

Written homework is graded both on the correctness of your work and on the clarity of your explanations. That is, you cannot get full credit unless you explain your answer. Sometimes a very simple explanation will do (see the first example that follows), and sometimes you need to explain more (see the second example). In any case, while a sequence of equations may be part of your answer, at least a little explanation of what those equations are all about is necessary.

We will not grade on grammar and spelling (unless your errors are so egregious that they make your explanation hard to understand), but we will grade on whether you use clear and understandable English.

In particular, you should begin by restating the problem you are solving. Don't just say "(34.)" and jump into the solution. A grader should be able to understand your solution completely without reference to the textbook, and you cannot understand a solution if you don't know what problem is being solved.

A few other things are necessary to get full credit on written homework: Write legibly. (If you do not have exemplary handwriting, writing large enough, and including enough white space on your paper, can help a lot.) Include your name. Use standard $8\frac{1}{2} \times 11$ paper, with no ragged edges. If you use more than one sheet, staple them together, and put your name on every sheet just in case. If you use the back sides of scrap paper, cross out the other sides in an obvious fashion.

Make sure to acknowledge anyone you worked with and any person or source you consulted. An informal note ("I worked with George Perez and Lydia Jones," or "I got help with Problem 45 in tutorial,") is fine. Working together is strongly encouraged.

An Example of a Problem Requiring Minimal Explanation

(54.) Find

$$\int x \sin x \, dx.$$

Solution: Use integration by parts:

$$u = x \qquad dv = \sin x dx$$

$$du = dx \qquad v = -\cos x$$

$$\int x \sin x \, dx = \int u \, dv = uv - \int v \, du =$$

$$x(-\cos x) - \int (-\cos x) \, dx = -x \cos x + \int \cos x \, dx =$$

$$\boxed{-x \cos x + \sin x + C}.$$

Note: Notice that every step of the computation, including the application of the formula for integration by parts, is shown.

An Example of a Problem Requiring More Explanation

(67.) A particle that moves along a straight line has velocity

$$v(t) = t^2 e^{-t}$$

meters per second after t seconds. How far will it travel during the first t seconds?

Solution: Let the distance the particle travels during the first t seconds be $d(t)$ meters. We want to find $d(t)$.

Because velocity is the derivative of distance, we know

$$d'(t) = v(t) = t^2 e^{-t}.$$

To find d, we need to integrate v:

$$\int v(t)\,dt = \int t^2 e^{-t}\,dt.$$

Evaluate this integral using integration by parts:

$$u = t^2 \qquad dv = e^{-t}\,dt$$

$$du = 2t\,dt \qquad v = -e^{-t}$$

$$\int t^2 e^{-t}\,dt = \int u\,dv = uv - \int v\,du =$$

$$t^2(-e^{-t}) - \int -e^{-t}(2t)\,dt = -t^2 e^{-t} + 2\int te^{-t}\,dt$$

We evaluate $\displaystyle\int te^{-t}\,dt$ using another application of integration by parts:

$$u = t \qquad dv = e^{-t}\,dt$$

$$du = dt \qquad v = -e^{-t}$$

$$\int te^{-t}\,dt = \int u\,dv = uv - \int v\,du =$$

$$t(-e^{-t}) - \int -e^{-t}\,dt = -te^{-t} + \int e^{-t}\,dt = -te^{-t} - e^{-t} + C.$$

Plugging this back in:

$$\int t^2 e^{-t}\,dt = -t^2 e^{-t} + 2\int te^{-t}\,dt = -t^2e^{-t} + 2\left(-te^{-t} - e^{-t}\right) + C =$$

$$-t^2e^{-t} - 2te^{-t} - 2e^{-t} + C.$$

This gives us

$$d(t) = -t^2e^{-t} - 2te^{-t} - 2e^{-t} + C,$$

for some constant C. To find the value of C, we use the fact that after 0 seconds the particle has traveled 0 meters, so $d(0) = 0$:

$$d(0) = 0$$

$$-0^2e^{-0} - 2(0)e^{-0} - 2e^{-0} + C = 0$$

$$-2 + C = 0$$

$$C = 2$$

$$\boxed{d(t) = -t^2e^{-t} - 2te^{-t} - 2e^{-t} + 2}.$$

Note: You can also do this problem by using the fact that the distance traveled between times $t = a$ and $t = b$ is the integral of the velocity function over the interval $[a, b]$.[1] Then $d(u)$ is the distance traveled between times $t = 0$ and $t = u$, so

$$d(u) = \int_0^u v(t)\,dt = \int_0^u t^2e^{-t}\,dt.$$

Computing the indefinite integral in the same way as above, we get

$$d(u) = \left(-t^2e^{-t} - 2te^{-t} - 2e^{-t}\right)\Big|_{t=0}^{t=u} = -u^2e^{-u} - 2ue^{-u} - 2e^{-u} + 2.$$

Rewriting in terms of t, we have

$$\boxed{d(t) = -t^2e^{-t} - 2te^{-t} - 2e^{-t} + 2}.$$

[1] See the discussion of the Net Change Theorem on page 324 of the textbook for an explanation of this.