1. Let $f(x, y) = 5 + 3x^2 + 3y^2 + 2y^3 + x^3$.

   (a) Final all critical points of f.

   (b) Use the second derivatives test to classify the critical points you found in (a) as a local maximum, local minimum, saddle point, or N/A if the test is not does not yield any information.

   (c) Determine whether or not $f(x, y)$ has a global maximum in the xy-plane. Justify your answer.

2. Suppose that as you move away from the point $(2, 0, 1)$, the function $f(x, y, z)$ increases most rapidly in the direction of the vector $3\hat{\imath} - \hat{\jmath} + 5\hat{k}$ and the rate of increase of f in this direction is 7. What is $\nabla f(2, 0, 1)$?

3. Consider the function $f(x, y) = x^3 + y^3 - 9xy + 1$

   (a) Find and classify the critical points on $f(x, y)$.

   (b) Find the absolute maximum and minimum values of $f(x, y)$ in the triangle with vertices $(0, 4)$, $(4, 0)$, and $(4, 4)$.

4. (a) Find a set of parametric equations for the line passing through the point $(2, 1, -1)$ and normal to the tangent plane of

$$4x + y^2 + z^3 = 8$$

(b) Suppose that $z = f(x, y)$, where $x = e^t$ and $y = t^2 + 3t + 2$. Given that $\partial z/\partial x = 2xy^2 - y$ and $\partial z/\partial y = 2x^2y - x$, find dz/dt when $t = 0$.

(c) Let $f(x, y) = (x - y)^3 + 2xy + x^2 - y$. Find the linear approximation $L(x, y)$ near the point $(1, 2)$.

5. Find the equation of the tangent plane to the surface $z = 4x^3y^2 + 2y$ at the point $(1, -2, 12)$.

6. Find all points at which the surface

$$x^2 + y^2 + 2x = z^2$$

has a vertical tangent plane.

7. Suppose that $f(x, y)$ is the distance between the origin $(0, 0)$ and the point (x, y). Use what you know about the significance of the gradient to find

$$\nabla f(3, 4)$$

without finding a formula for f or computing any derivatives.

8. Find a function parametrizing the curve γ given by the intersection of the plane with equation $z = x - y$ and the cylinder with equation $x^2 + y^2 = 1$.

   Find an equation for the line tangent to γ at the point where $(x, y) = (0, 1)$.

   Represent the arclength of γ as an integral. You do NOT have to evaluate this integral; just write it down.