MATH 8 PRACTICE EXAM 1, WINTER 2004

Disclaimer: This set of problems is meant to indicate neither the length nor composition of the actual exam. It consists of problems that we rejected in creating the actual exam. These problems are meant to help you practice for the exam, but are in no way a comprehensive set of problems. You may well see completely different types of problems on the exam.

1. Long answer

Problem 1.1. Solve the initial value problem

$$y' + 2y = x^2, \qquad y(0) = 0.$$

Problem 1.2. A patient receives glucose intravenously at rate r and the glucose is used by the body at rate $kx(t)$, where $x(t)$ is the concentration of glucose in the bloodstream at time t and $k > 0$ is a constant.

(a) Write a differential equation which describes the change in glucose concentration in the blood.
(b) Find the general solution to the above differential equation.
(c) Suppose the inital glucose concentration in the blood is c_0. What is the glucose concentration at time t?

Problem 1.3. Let $f(x) = \ln(x)$.

(a) Find the Taylor polynomial $T_3(x)$ of degree 3 for f centered at $a = 1$.
(b) Use T_3 to estimate $\ln(\frac{3}{2})$.
(c) Find an upper bound for the absolute value of the error in approximating $\ln(x)$ by T_3 for x in the interval $[\frac{1}{2}, \frac{3}{2}]$. That is, find a number E such that $|R_3(x)| \leq E$ for all $\frac{1}{2} \leq x \leq \frac{3}{2}$.

Problem 1.4. Consider the series

$$\pi - e + \frac{e^2}{\pi} - \frac{e^3}{\pi^2} + \cdots$$

If it diverges, explain why. If it converges, explain why.

2. True or False

Problem 2.1. One of the seven complex 7th roots of 4 is a pure imaginary number (i.e. has real part equal to 0).

Problem 2.2. The modulus of $(1+3i)(1-\frac{1}{4}i)$ is less than 1.

Problem 2.3. Let $a_n = \frac{1}{n+6}$. Then $\lim_{n\to\infty} a_n = 0$.

Problem 2.4. Let $a_n = 1 - \left(\frac{1}{2}\right)^n$. Then the series $\sum_{n\geq 0} a_n$ converges.

Problem 2.5. The general solution of the differential equation $y - 4\frac{y'}{y} = 4$ is $y = C_1e^{-2x} + C_2xe^{-2x}$.

Problem 2.6. Let $a_n = \left(\frac{1}{5}\right)^n$. Then $\sum_{n=2}^{\infty} a_n = \frac{1}{20}$.

3. Multiple Choice

Problem 3.1. The power series $\sum_{n=0}^{\infty}(3x+2)^n$ converges for

A. $-\frac{2}{3} < x < \frac{2}{3}$
B. $-1 < x < -\frac{1}{3}$
C. $-1 < x < 1$
D. $-\frac{1}{3} < x < \frac{1}{3}$
E. None of these

Problem 3.2. The general solution of $(y-1)\frac{dy}{dx} = 2x$, $y > 1$ is of the form

A. $y = \sqrt{2x^2} + C$
B. $y = \sqrt{2x^2+1} + C$
C. $y = \sqrt{2x^2+C} + 1$
D. $y = \sqrt{2(x+1)^2} + C$
E. None of these

Problem 3.3. The radius of convergence of the power series $\sum_{n=0}^{\infty} \frac{nx^n}{5^n}$ is

A. 0
B. $\frac{1}{5}$
C. 5
D. ∞
E. None of these

Problem 3.4. The value of $z = \frac{(1+2i)(2-3i)}{(2-i)(3+2i)}$ is

A. 1
B. -1
C. i
D. -i
E. None of these

Problem 3.5. Which of the following differential equations has a general solution of the form $y = c_1e^{-4t} + c_2te^{-4t}$?

A. $y'' + 16y = 0$
B. $y'' - 8y' + 16y = 0$
C. $y'' + 8y' + 16y = 0$
D. $y'' - 16y = 0$
E. $y'' + 8y' - 16y = 0$

Problem 3.6. The sum of the series $\frac{3}{7} - \frac{3}{14} + \frac{3}{28} - \frac{3}{56} + \cdots$ is given by

A. $\frac{2}{7}$
B. $\frac{7}{2}$
C. $\frac{15}{56}$
D. $\frac{1}{7}$
E. None of these

Problem 3.7. A spring submerged in a liquid has length 5m when holding a mass of 3 kg. A force of 50 N is required to stretch the spring to a length of $5\frac{2}{3}$ m. The damping constant is 12. If $y(t)$ is the displacement of the mass from equilibrium at time t, then y satisfies the differential equation

A. $y'' - 12y' + 40y = 0$
B. $y'' + 4y' + \frac{40}{3}y = 0$
C. $y'' + 4y' + 20y = 0$
D. $y'' + 4y' + 25y = 0$
E. $y'' + 12y' + 60y = 40$