Math 8 Practice Exam Problems

Disclaimer: These problems primarily from the material since the last exam. You should also consult your previous exams and the previous practive exam problems.

1. Find the general solution to the differential equation $\dfrac{dy}{dx} + \dfrac{y}{x \ln x} = x$.

2. Find the equation of the tangent plane to the level surface of $f(x, y, z) = ye^{-x^2} \sin z$ at $(0, 1, \pi/3)$.

3. Suppose that $z = f(x, y)$ is a smooth real-valued function of two variables, and that $\dfrac{\partial f}{\partial x}(1, 1) = 3$ and $\dfrac{\partial f}{\partial y}(1, 1) = -1$. If $x = s^2$ and $y = s^3$, we may then view z as a function of the single variable s. The value of $\dfrac{dz}{ds}$ at $s = 1$ is

4. Find an equation of the curve $y = f(x)$ that passes through the point $(1, 1)$ and intersects all level curves of the function $g(x, y) = x^4 + y^2$ at right angles.

5. A ball is placed at the point $(1, 2, 3)$ on the surface $z = y^2 - x^2$. Give the direction in the xy-plane corresponding to the direction in which the ball will start to roll. Describe the path in the xy-plane which the ball will follow.

6. Let $f(x, y) = x^4 + y^4 + x^2 - y^2$. Find and classify all critical points of f. Use the method of Lagrange multipliers to find the largest and smallest values of f on the circle $x^2 + y^2 = 4$.

7. Consider $\lim\limits_{(x,y)\to(0,0)} \dfrac{x^2 - y^2}{x^2 + y^2}$. Does the limit exist? Why or why not?

8. The temperature at the point (x, y, z) is given by $T(x, y, z) = xy^2z$. Find the direction of maximum increase in temperature at the point $(1, -2, 3)$. If you move so that your velocity as you pass through the point $(1, -2, 3)$ is $(1, 2, 2)$, then what is the rate of temperature increase as you pass through $(1, -2, 3)$?