Practice Problems for Exam II
Math 8
February 11, 2000

DISCLAIMER: This set of problems is not meant to indicate length or the actual composition of the exam. Each problem could have appeared on the exam, but was rejected for one reason or another. I hope they will provide some help in studing for the exam.

1. Consider a cube whose sides have length 1. Suppose that one vertex $P = (a, b, c)$ and the diametrically opposite vertex $Q = (a + 1, b + 1, c + 1)$. Find the angle between the vector from P to Q and any edge which has P as a vertex.

2. A straight river 500m wide flows east at a constant speed of 3 km/h. If you can row your boat at a speed of 5km/h in still water, in what direction should you head if you wish to row from a point A on the south shore to a point B on the north shore directly north of A. How long will the trip take?

3. (a) If $\mathbf{u}$ and $\mathbf{v}$ are vectors in 3-space ($\mathbb{R}^3$), with $\mathbf{u} \bullet \mathbf{v} = \sqrt{18}$, $\mathbf{u} \bullet \mathbf{u} = 6$ and $\mathbf{v} \bullet \mathbf{v} = 9$, find the length of $\mathbf{u} \times \mathbf{v}$.

   (b) Let $\mathbf{u} = (1, -5, 3)$ and $\mathbf{v} = (1, 1, 1)$. Find the vector projection of $\mathbf{u}$ in the direction of $\mathbf{v}$, that is find the vector $\mathbf{u}_\mathbf{v} = \mathrm{Proj}_\mathbf{v}\mathbf{u}$.

   (c) Find an equation of the plane which contains the two lines $\mathbf{r} = (2 + 2t, 1 - 3t, 4 + t)$ and $\mathbf{r} = (1 - 2s, 3s, 2 - s)$.

4. Find an equation of the line through the point $(1, 2, 3)$ perpendicular to the plane $x - 3y + 5z = 1$.

5. Are the lines $\mathbf{x} = (1, 2, 3) + t(4, -4, 6)$ and $\mathbf{r} = (3 + s, 2s, 6 + 3s)$ skew, parallel, or intersecting?

6. Consider the matrix $A = \begin{pmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 2 \end{pmatrix}$. Show that $A\mathbf{x} = \mathbf{b}$ is solvable for all $\mathbf{b} \in \mathbb{R}^2$, and find all solutions to $A\mathbf{x} = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$. What is the dimension of the solution space of $A\mathbf{x} = \mathbf{0}$?

7. Find the volume of the parallelepiped determined by the vectors $\mathbf{u} = (1, 2, 3)$, $\mathbf{v} = (2, 0, 1)$, and $\mathbf{w} = (3, 0, 4)$.

8. An aircraft flies 200 kph in still air. There is a wind from the north at 100kph. The pilot wants to fly due east. In what direction should the pilot fly, and what is the ground speed of the aircraft?

9. Find the inverse of the matrix $A = \begin{pmatrix} 1 & 2 \\ 2 & 5 \end{pmatrix}$ and use it to solve the system $A\mathbf{x} = \begin{pmatrix} 3 \\ 4 \end{pmatrix}$.