Practice Problems for Exam I
Math 8
January 26, 2000

DISCLAIMER: This set of problems is not meant to indicate length or the actual composition of the exam. Each problem could have appeared on the exam, but was rejected for one reason or another. I hope they will provide some help in studing for the exam.

1. Find the general solution to the differential equation $(D-3)^3(y)=0$.

2. Find the general solution to $\dfrac{d^2y}{dx^2}+9y=0$. Find the solution which satisfies $y(0)=3$ and $y'(0)=6$.

3. Find the solutions to the following differential equations:

   (a) Find the solution of $\dfrac{dy}{dx}=\dfrac{\ln x+1}{y}$ which passes through the point $(1,2\sqrt{2})$.

   (b) Find the general solution of $\dfrac{dy}{dx}=\dfrac{3x(1+y^2)}{2y}$.

   (c) Find the general solution to $\dfrac{dy}{dx}-3x^2y=x^2$.

4. Write the complex number $(-2\sqrt{3}-2i)^{13}$ in the form $r(\cos\theta+i\sin\theta)$ for $\theta\in[0,2\pi)$.

5. Consider the differential equation $\dfrac{d^2y}{dx^2}+b\dfrac{dy}{dx}+cy=0$ in which b and c are positive constants satisfying $b=2\sqrt{c}$. Determine the general solution of this differential equation.

6. The rate at which students forget formulas is proportional to the number of formulas $(\dfrac{dF}{dt}=kF)$. If 10 minutes after walking into an exam, the student remembers only a third of the formulas initially stored in short-term memory, what fraction of the formulas are remembered 30 minutes into the exam.

7. Let $f(x) = \cos x$. Use Taylor's polynomial $P_6(x)$ of degree 6 about $x = 0$ to estimate $\cos 2$. Find an upper bound for the absolute value of the error in your estimate. (How do $P_6(x)$ and $P_7(x)$ differ? Can you use this to get a smaller upper bound for the error in your estimate?) Without knowing the value of $\cos 2$, can you tell if your estimate is bigger or smaller than $\cos 2$?

8. (Amended version) A container with a capacity of 5000 liters contains 1000 liters of brine solution with 20 kg of dissolved salt. Into the tank flows a brine solution with a concentration of .03 kg/liter of salt at a rate of 20 liters/min. In addition, pure water flows into the tank at a rate of 20 liters/min. The mixture in the tank is constantly mixed and flows out of the container at a rate of 30 liters/min. Find the differential equation and initial condition satisfied by the number $S(t)$ of kg of salt in the container at time t minutes. What integrating factor would you use to solve this equation? (Writing an explicit formula for $S(t)$ is algebraically tedious, so you shouldn't bother unless your curious.)