Math. 8, Spring 1999

PRACTICE FINAL EXAM

1. Evaluate the integrals:

(a) $\int x^2 \cos x dx$

(b) $\int \frac{\cos x}{\sin^2 x} dx$

(c) $\int \frac{x^2 + 2x}{(x+1)^2} dx$

2. A certain population satisfies the growth law $\frac{dP}{dt} = \frac{2t}{3}$, where t is measured in years after January 1, 1990. On January 1, 1990, there were 500 people. How many will there be on January 1, 2000?

3. Determine the interval of convergence of the power series

(a) $\sum_{n=1}^{\infty} \frac{5^n}{\sqrt{n}} x^n$

(b) $\sum_{n=0}^{\infty} \frac{3^n x^n}{n!}$

4. Find the sum of the series and state the interval on which this sum is valid

(a) $\sum_{n=0}^{\infty} \frac{(-1)^n x^{4n}}{n!}$

(b) $\sum_{n=0}^{\infty} \frac{(x+1)^{2n}}{9^n}$

5. Use Maclaurin series to find $f^{(10)}(0)$ for

(a) $f(x) = x^7 e^{3x}$

(b) $f(x) = \ln(1 - 3x)$

6. Points $P(1, 1, 2)$, $Q(1, 0, 1)$ and $R(-1, -1, 0)$ determine a plane T.

(a) Find a unit vector which is orthogonal to the plane T.

(b) Find a vector equation of the line passing through $(4, 0, -5)$ that is perpendicular to the plane T.

7. Use vectors to show that the line joining the midpoints of two sides of a triangle is parallel to the third side and is half as long.

8. Show that the lines $\mathbf{r_1} =< 2 - t, 4 + 2t, -3 + 4t >$ and $\mathbf{r_2} =< 1 + t, 5 - 3t, 3 - 2t >$ intersect, and find an equation of the plain that contains them.

9. Solve the following differential equations.

(a) $xy' + y = \frac{1}{x}, \quad y(1) = 2$

(b) $y''' + 4y' = 0$

10. Show that the region under the curve $y = 1/x$ on the interval $[1, \infty)$ has infinite area, but that the solid obtained by rotating this region about the x-axis has finite volume.