

SECTION 15.6 THE GRADIENT VECTOR FIELD

Problem 32, page 971.

You are climbing a hill whose shape is given by the equation

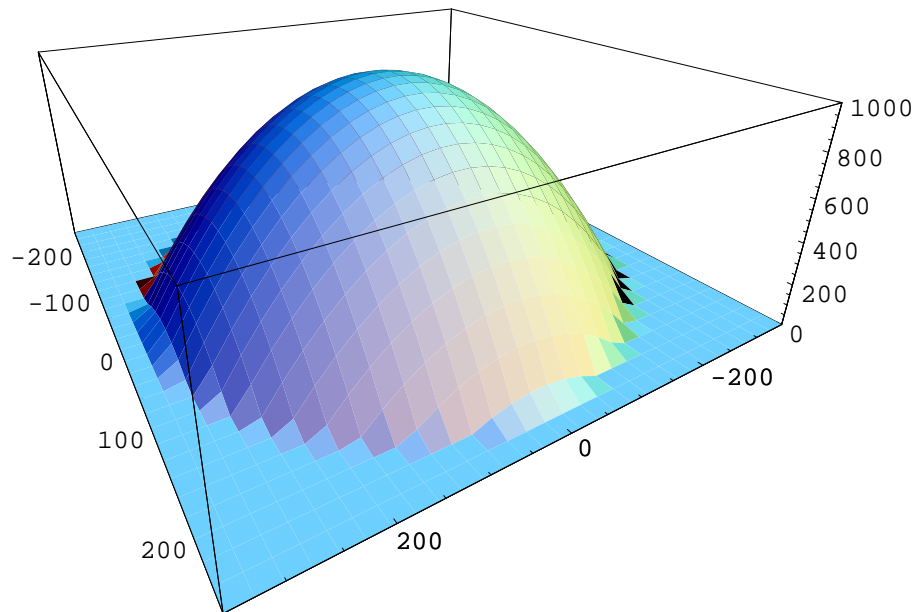
$$z = 1000 - 0.01x^2 - 0.02y^2$$

and you are standing at the point with coordinates $(60, 100, 764)$. In which direction should you proceed initially in order to reach the top of the hill fastest? Draw the level curves and the gradient.

A graphical approach.

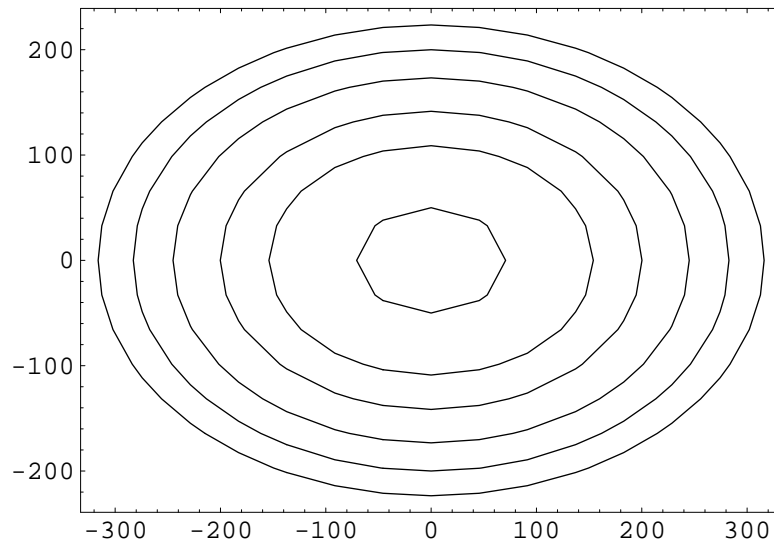
We start with the graph of the hill :

```
Plot3D[1000 - 0.01 x^2 - 0.02 y^2, {x, -350, 350}, {y, -250, 250},  
PlotRange -> {0, 1000}, ViewPoint -> {0.6, 1, 0.5}, PlotPoints -> 30, Mesh -> False  
]
```



The following two pictures show some of the level curves, including the one at height $z = 764$, which contains the point given in the statement of the problem, and the gradient vector field.

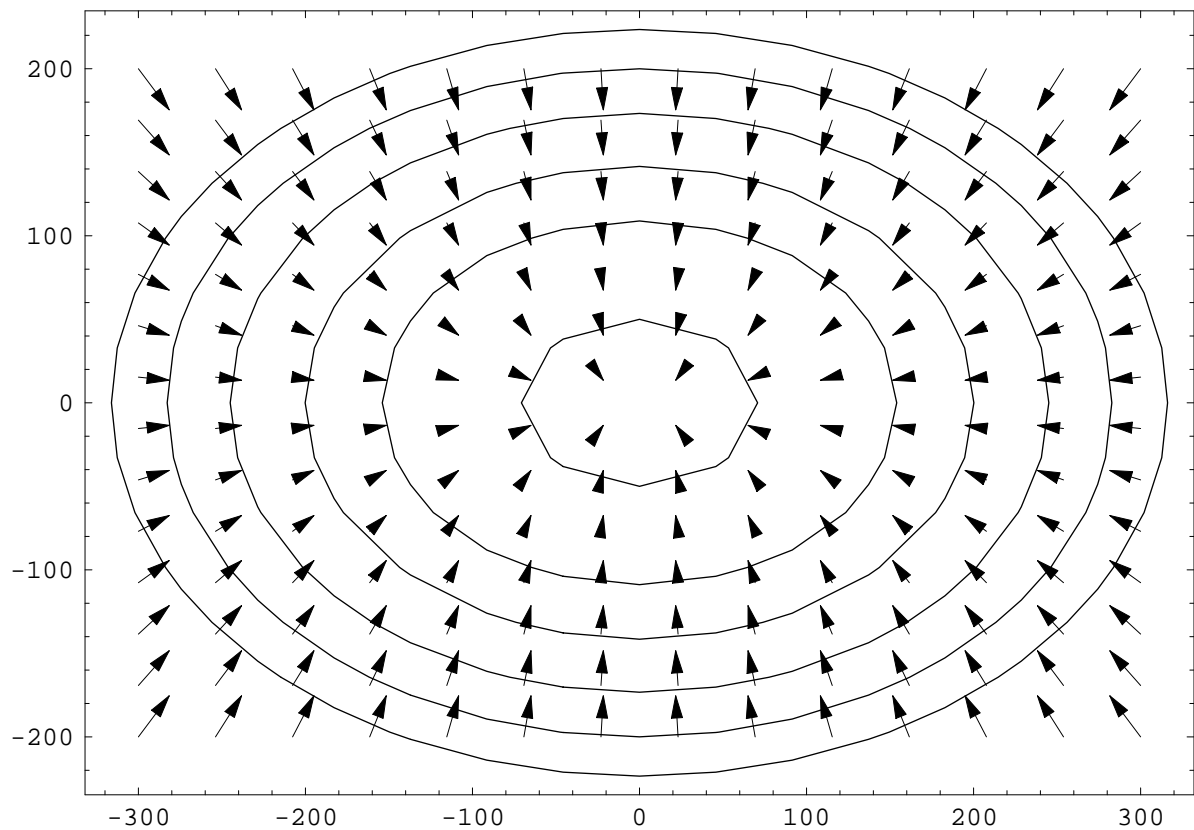
```
g1 = ContourPlot[1000 - 0.01 * x^2 - 0.02 * y^2, {x, -320, 320}, {y, -230, 230},  
Contours -> {0, 200, 400, 600, 764, 950}, ContourShading -> False,  
AspectRatio -> Automatic]
```



```
<< Graphics`
```

```
g2 = PlotVectorField[{-0.02 * x, -0.04 * y}, {x, -300, 300}, {y, -200, 200},  
AspectRatio -> Automatic, PlotPoints -> 14]
```

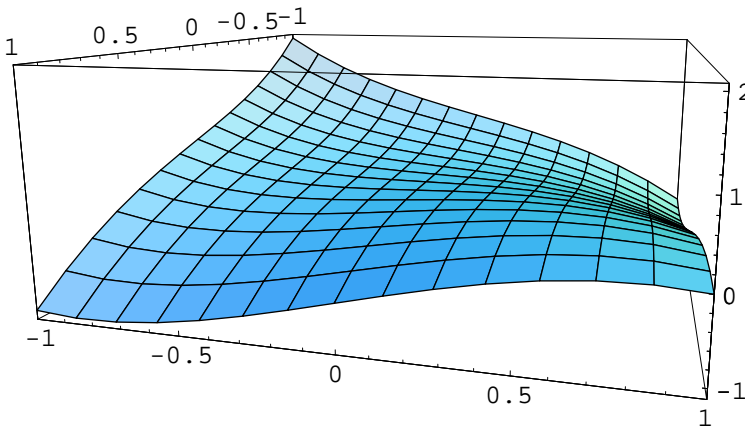
```
Show[g1, g2, AspectRatio -> Automatic]
```



■ **EXAMPLE (to play with and test your understanding about the gradient vector field!)**

The graph and some level curves the function $f(x, y) = xy - 0.5x^3 - 0.5y^3$ are given below. Draw the gradient vector field of f .

```
Plot3D[x*y - 0.5*x^3 - 0.5*y^3, {x, -1, 1}, {y, -1, 1},
ViewPoint -> {1.5, 0.4, 0.3}]
```



```
ContourPlot[x*y - 0.5*x^3 - 0.5*y^3, {x, -1, 1}, {y, -1, 1},
Contours -> 40, ContourShading -> False]
```

