Assignment 18: Matrix equations and matrix algebra

1. For the two matrices below, find the dimension of the solution space of $A\mathbf{x} = \mathbf{0}$. Then find all the solutions. You should be able to determine the dimension without finding the solutions.
$$A = \begin{pmatrix} 1 & 2 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix} \text{ and } A = \begin{pmatrix} 1 & 2 & 0 & 0 & 3 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 2 \end{pmatrix}$$

2. Suppose that the matrix A has row-reduced echelon form R given below.
$$A = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 6 & 7 & 8 & 9 & 1 \\ 2 & 3 & 4 & 5 & 6 \\ 7 & 8 & 9 & 1 & 2 \end{pmatrix} \text{ and } R = \begin{pmatrix} 1 & 0 & -1 & 0 & 0 \\ 0 & 1 & 2 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$
Find all solutions to the matrix equation $A\mathbf{x} = \begin{pmatrix} 6 \\ 7 \\ 8 \\ 9 \end{pmatrix}$ given that $\mathbf{x} = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}$ is a particular solution.

3. Let $A = \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{pmatrix}$, $B = \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{pmatrix}$, and $C = \begin{pmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{pmatrix}$.
Determine which of the nine products $A^2, AB, AC, BA, B^2, BC, CA, CB, C^2$ are defined. Evaluate the first three valid products.

4. Find $\begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}^{1234}$. You may want to find $\begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}^2$ and $\begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}^3$ to start. Can you prove your result?

5. Find two 2×2 matrices A and B neither of which is the zero matrix, but for which $AB = \mathbf{0}$.