Math 8 Practice Exam Problems

Disclaimer: A few problems from the recent material

1. Find the general solution to the differential equation $\dfrac{dy}{dx} + \dfrac{y}{x \ln x} = x$.

2. Find the equation of the tangent plane to the level surface of $f(x, y, z) = ye^{-x^2} \sin z$ at $(0, 1, \pi/3)$.

3. Suppose that $z = f(x, y)$ is a smooth real-valued function of two variables, and that $\dfrac{\partial f}{\partial x}(1, 1) = 3$ and $\dfrac{\partial f}{\partial y}(1, 1) = -1$. If $x = s^2$ and $y = s^3$, we may then view z as a function of the single variable s. The value of $\dfrac{dz}{ds}$ at $s = 1$ is

4. Find an equation of the curve $y = f(x)$ that passes through the point $(1, 1)$ and intersects all level curves of the function $g(x, y) = x^4 + y^2$ at right angles.

5. A ball is placed at the point $(1, 2, 3)$ on the surface $z = y^2 - x^2$. Give the direction in the xy-plane corresponding to the direction in which the ball will start to roll. Describe the path in the xy-plane which the ball will follow. At the point $(1, 2, 3)$ what is the maximum rate at which the ball is descending.

6. Let $f(x, y) = x^4 + y^4 + x^2 - y^2$. Find and classify all critical points of f. Use the method of Lagrange multipliers to find the largest and smallest values of f on the circle $x^2 + y^2 = 4$.

7. Consider a function $z = f(x, y)$ which is defined an has partial derviatives of all orders for all x and y. Suppose the function $f(x, b)$ has a local maximum at $x = a$ and the function $f(a, y)$ has a local minimum at $y = b$. Can one infer that the point (a, b) is a critical point, saddle point, local maximum, local minimum?