Math 8 Practice Exam Problems

Disclaimer: This set of problems is meant neither to indicate the length nor composition of the actual exam. These are merely problems which were considered for inclusion on your exam, but for one reason or another were rejected. On the other hand, they should provide some flavor of the type of problems we considered.

1. Find the Taylor series (about $x = 0$) for $f(x) = \begin{cases} \sin x/x & \text{if } x \neq 0 \\ 1 & \text{if } x = 0 \end{cases}$

2. Find the radius and endpoints of the interval of convergence of the power series $\displaystyle\sum_{n=0}^{\infty} \frac{n(x-3)^n}{7^n}$. Note the endpoints may not actually be in the interval of convergence.

3. $\displaystyle\sum_{n=10}^{\infty} \left(\frac{-2}{3}\right)^n = ?$

4. Find an equation of the line through the point $(1, 2, 3)$ orthogonal to the plane $x - 3y + 5z = 1$.

5. Are the lines $\mathbf{x} = \langle 1, 2, 3 \rangle + t\langle 4, -4, 6 \rangle$ and $\mathbf{r} = \langle 3 + s, 2s, 6 + 3s \rangle$ skew, parallel, or intersecting?

6. Consider the matrix $A = \begin{pmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 2 \end{pmatrix}$.

   Show that $A\mathbf{x} = \mathbf{b}$ is solvable for all $\mathbf{b} \in \mathbb{R}^2$, and find all solutions to $A\mathbf{x} = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$. What is the dimension of the solution space of $A\mathbf{x} = \mathbf{0}$?

7. Find the volume of the parallelopiped determined by the vectors $\mathbf{u} = \langle 1, 2, 3 \rangle$, $\mathbf{v} = \langle 2, 0, 1 \rangle$, and $\mathbf{w} = \langle 3, 0, 4 \rangle$.

8. An aircraft flies 200 kph in still air. There is a wind from the north at 100kph. The pilot wants to fly due east. In what direction should the pilot fly, and what is the groundspeed of the aircraft?

9. Find the inverse of the matrix $A = \begin{pmatrix} 1 & 2 \\ 2 & 5 \end{pmatrix}$ and use it to solve the system $A\mathbf{x} = \begin{pmatrix} 3 \\ 4 \end{pmatrix}$.