LECTURE OUTLINE

Absolute and Conditional Convergence

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Math 8

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Goals

Remainder Estimates for Alternating Series

Absolute Convergence

Conditional Convergence

Warm Up

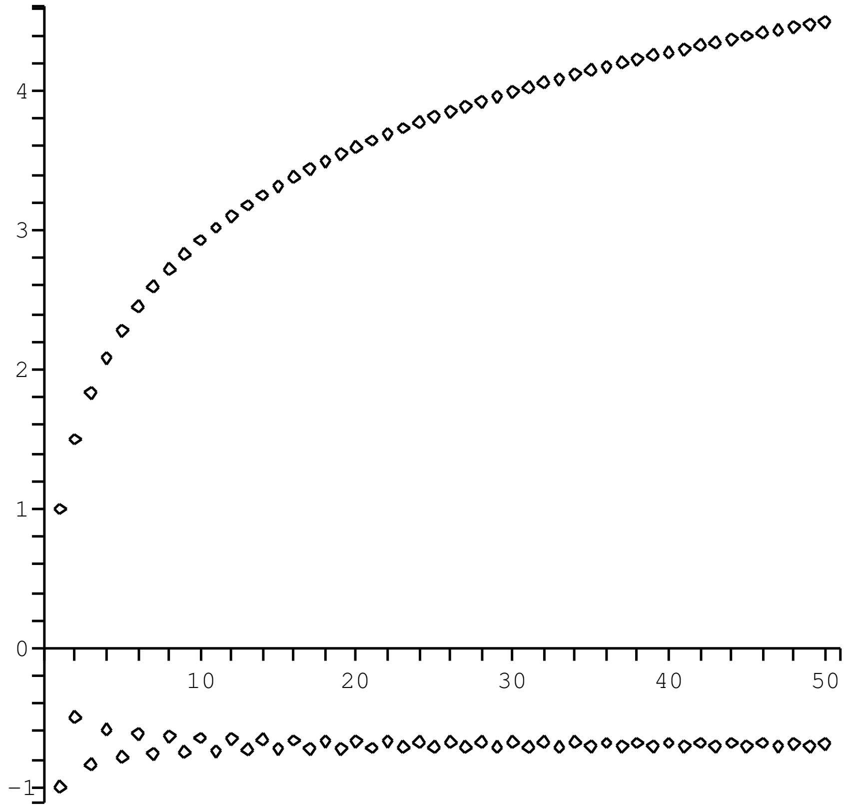
Find where $\sum \frac{f(n)}{n^p}$ converges and explain why when

$$f(n) = 1$$

and when

$$f(n) = (-1)^n.$$

$$\sum_{n=1}^{\infty} \frac{(-1)^n}{n} \text{ verses } \sum_{n=1}^{\infty} \frac{1}{n}$$



Alternating Series

An *alternating series* is one whose terms are alternately positive and negative. In other words, $\sum(-1)^{n-1}b_n$ with $b_n > 0$.

Alternating Series Test: If $b_{n+1} \leq b_n$ and $\lim_{n\to\infty} b_n = 0$, then the alternating series is convergent.

Ex: Is $\sum_{n=1}^{\infty} \frac{(-1)^{n-1}\ln(n)}{\sqrt{n}}$ convergent?

Remainder Estimate

Given a series $\sum a_k$ for each n we may express our series as $\sum a_i = s_n + R_n$. We call $R_n = \sum_{k=n+1}^{\infty} a_k$ the *remainder*.

Alternating Series Remainder Estimate: Given a convergent alternating series

$$|R_n| \leq b_{n+1}.$$

Ex: How big will n need to be so that s_n is with in 0.01 of $\sum \frac{(-1)^{n-1}}{\sqrt{n}}$.

Absolute Convergence

A series $\sum a_n$ is called *absolutely convergent* if the series $\sum |a_n|$ is convergent.

Theorem: If a series is absolutely convergent, then it is convergent.

Ex: Is $\sum_{n=1}^{\infty} \frac{(-1)^n(n^3+3n+2)}{\sqrt{n^9+7}}$ convergent?

Conditional Convergence

A series $\sum a_n$ is called *conditionally convergent* if the series $\sum a_n$ is convergent but $\sum |a_n|$ is divergent.

When is $\sum \frac{(-1)^n}{n^p}$ conditionally convergent .

In Between Alternating and Positive

Recall, we can factor a number into *distinct primes* raised to some *powers*. For example,

$$12 = (2)^2(3)$$

while

$$30 = (2)(3)(5).$$

We say that 30 has 3 *distinct factors* (2,3, and 5), while we say that 12 has a factor with a *power* bigger than 1 (namely the 2 is squared).

In Between

Let $f(n)$ be equal to the $Möbius$ *Function* which is

0 if some factor of n has a *power* bigger than 1,

$+1$ if the factors are distinct and there are an even number of them,

and -1 if the factors are distinct and there are an odd number of them.

The Challenge: Find where $\sum \frac{Möbius(n)}{n^p}$ converges.