LECTURE OUTLINE
The Comparison Test

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Math 8

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Goals

Comparison Test

Limit Comparison Test

Comparison Test

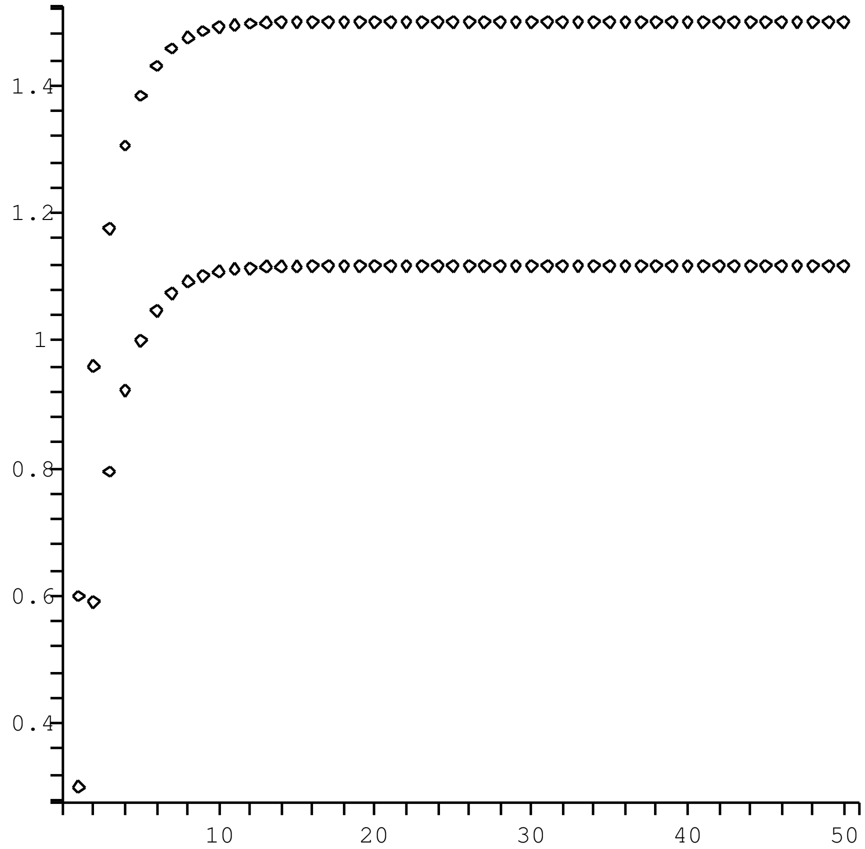
Suppose that $\sum a_n$ and $\sum b_n$ are series with positive terms.

(1) If $\sum b_n$ converges and $a_n \leq b_n$, then $\sum a_n$ converges.

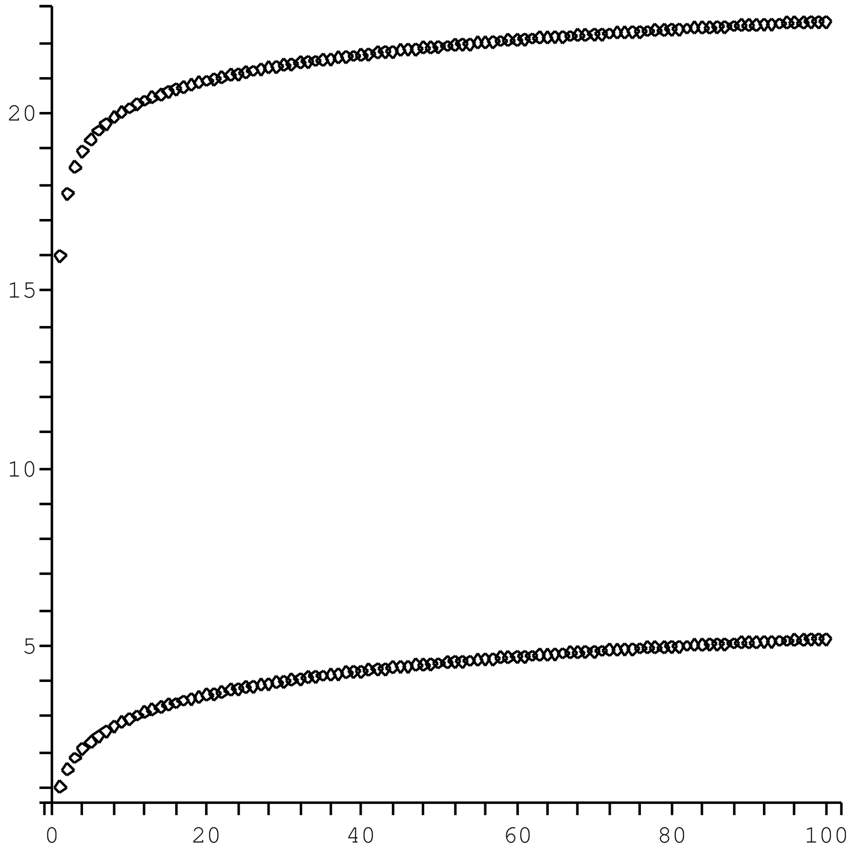
(2) If $\sum b_n$ is divergent and and $a_n \geq b_n$, then $\sum a_n$ is divergent.

Ex: Decide whether: $\sum_{n=1}^{\infty} \frac{3^n - 1}{5^n + n + 4}$ and $\sum_{n=1}^{\infty} \frac{n^2 + 2n + 5}{n^3 - (1/2)}$ are convergent or divergent.

r: $\sum_{n=1}^{\infty} \frac{3^n-1}{5^n+n+4}$



$$\sum_{n=1}^{\infty} \frac{n^2+2n+5}{n^3-(1/2)}$$



Real Numbers are not Nutzo

Monotonic Sequence theorem: Every bounded, monotonic sequence is convergent.

Limit Comparison Test

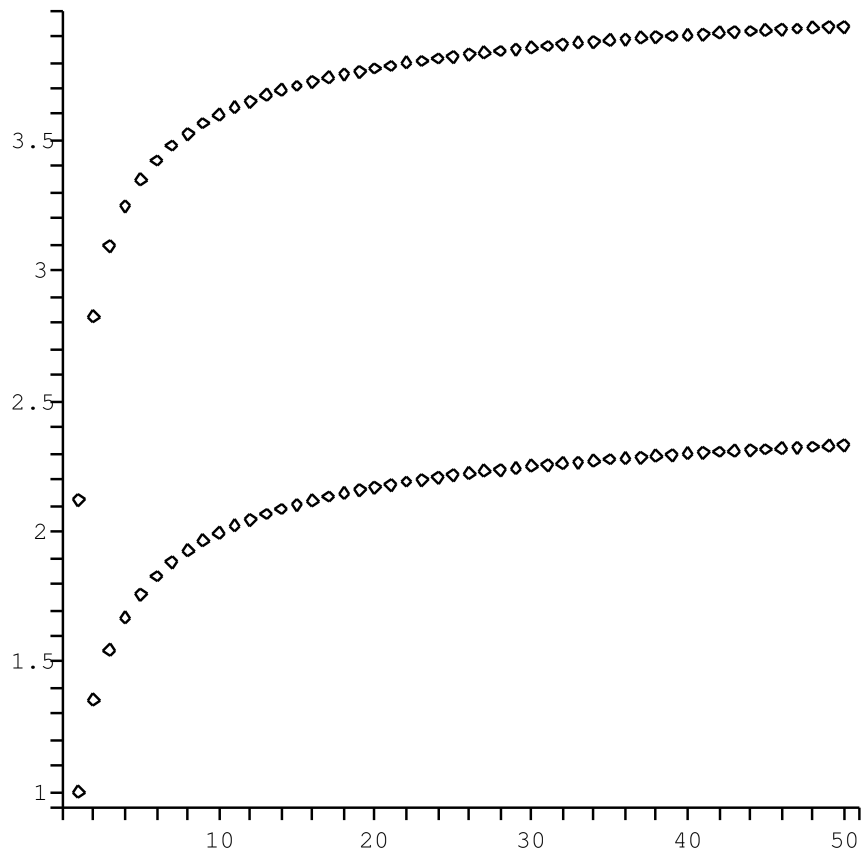
Suppose that $\sum a_n$ and $\sum b_n$ are series with positive terms and

$$\lim_{n\to\infty} \frac{a_n}{b_n} = c$$

where $0 < c < \infty$, then either both series converge or diverge.

Ex: Decide whether $\sum_{n=1}^{\infty} \frac{n^3+3n+2}{\sqrt{n^9+7}}$ is convergent or divergent.

$$\sum_{n=1}^{\infty} \frac{n^3+3n+2}{\sqrt{n^9+7}}$$



Alternating Series

An *alternating series* is one whose terms are alternately positive and negative. In other words, $\sum(-1)^{n-1}b_n$ with $b_n > 0$.

Alternating Series Test: If $b_{n+1} \leq b_n$ and $\lim_{n\to\infty} b_n = 0$, then the alternating series is convergent.

Ex: For which p does $\sum_{n=1}^{\infty} \frac{(-1)^n}{n^p}$ converge.

$$\sum_{n=1}^{\infty} \frac{(-1)^n}{n} \text{ verses } \sum_{n=1}^{\infty} \frac{1}{n}$$