LECTURE OUTLINE
Series

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Math 8

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Goals

Introduce Series

Geometric Series

Integral Comparison Test

A Series

A series is a new sequence $\{s_n\}$ built from an old sequence $\{a_n\}$ by letting

$$s_n = a_1 + \ldots + a_n = \sum_{i=1}^{n} a_i.$$

If the limit of $\{s_n\}$ exist we denote it as

$$\sum_{i=1}^{\infty} a_i = \sum a_i,$$

and say that $\{a_n\}'s$ sum is convergent. Otherwise we say the sum is divergent.

A Telescoping Sum

Let $a_n = \frac{1}{(n)(n+1)}$. Decide whether $\sum a_i$ is convergent, and if it is convergent compute $\sum a_i$.

The Simple Test for Divergence

Theorem: If a series $\sum a_n$ is convergent, then $\lim_{n\to\infty} a_n = 0$.

In other words, if $\lim_{n\to\infty} a_n$ is divergent or not equal to zero then, $\sum a_n$ must be divergent.

The Most Important Example

The Geometric Series: Let $a_n = r^n$. For which r is $\sum a_i$ is convergent? When $\sum a_i$ is convergent, compute $\sum a_i$.

Answer: The sum is convergent if and only if $|r| < 1$. When $|r| < 1$ we have $\sum_{i=1}^{\infty} a_i = \frac{r}{1-r}$. Notice, this implies $\sum_{i=0}^{\infty} r^i = \frac{1}{1-r}$.

Example

Express $0.429429429....$ as a ratio of integers.

The Integral Test

Theorem: Suppose f is continuos, positive, decreasing function on $[1, \infty)$ and let $a_n = f(n)$. Then $\sum a_i$ is convergent if and only if the improper integral $\int_1^\infty f(x)dx$ is convergent.

Example: Is $\sum \frac{1}{n^2}$ convergent?

Basic Rules

If $\sum a_n$ and $\sum b_n$ are convergent, then

$$\begin{aligned} \sum c a_n &= c \sum a_n \\ \sum (a_n + b_n) &= \sum a_n + \sum b_n. \end{aligned}$$

Example: Is $\sum \left(\frac{n-1}{n^3} + 3^{n+2}(1/4)^n \right)$ convergent?