LECTURE OUTLINE
Coordinates and vectors

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Math 8

Oct. 22, 2004

Goals

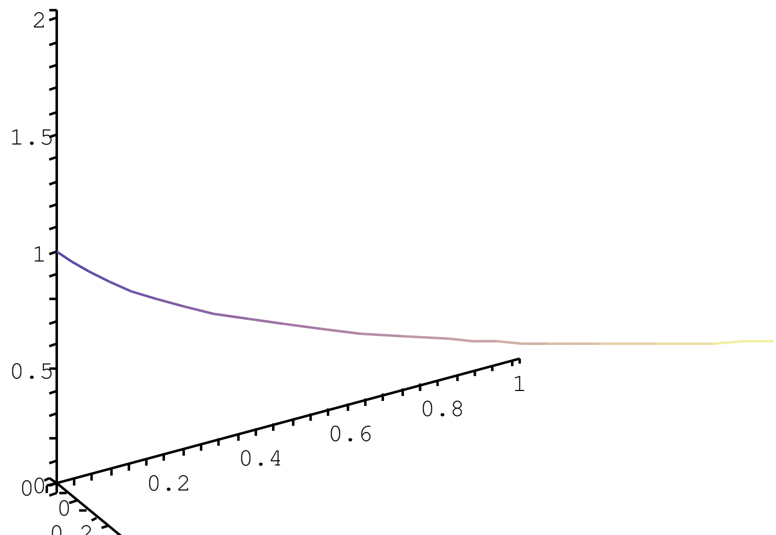
Introduce Coordinates

Vectors

Vector Addition

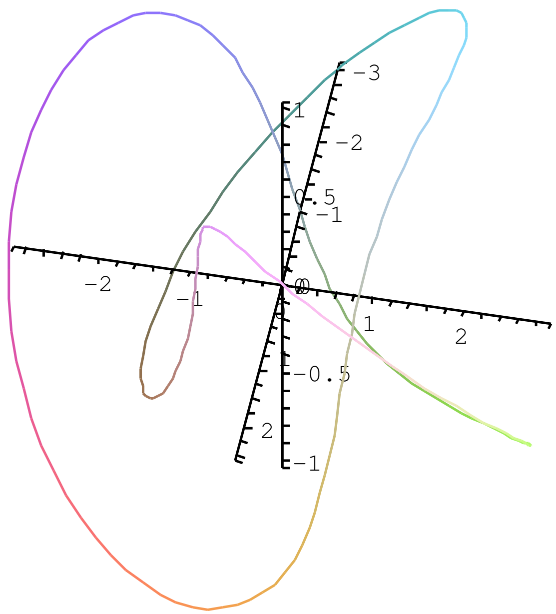
Coordinates

Introducing the (x, y, z) coordinates of three dimensional space.



Movement

Objects can "move" through these coordinates $(x(t), y(t), z(t))$.



Vector

We will also also want to encode direction in magnitude. For example, we will want to say go in direction "blah" for a distance of "blah". We encode such a statement with a *vector*,

$$\vec{v} = x\hat{i} + y\hat{j} + z\hat{k} = < x, y, z >$$

Norm

Given a vector $\vec{v} = x\hat{i} + y\hat{j} + z\hat{k}$ we encoded its *magnitude* (also *norm* or *length*) via

$$|\vec{v}| = \sqrt{x^2 + y^2 + z^2},$$

Ex: Let $\vec{a} =< -1, 2, 5 >$ and find $|\vec{a}|$.

Scalar Multiplication

To encode the vector's direction, we must first learn *scalar multiplication*:

$$c\vec{v} = cx\hat{i} + cy\hat{j} + cz\hat{k}$$

Notice, the norm satisfies

$$|c\vec{v}| = |c||\vec{v}|.$$

Direction

$\vec{v}$'s direction is given by

$$\hat{v} = \frac{\vec{v}}{|\vec{v}|}$$

$\hat{v}$ is called a *unit vector* and has norm 1, and is usually viewed as unitless.

Position Vector

In Euclidean space, once we've chosen a coordinate system we can view the point (x, y, z) as going from the origin in the direction and with the distance determined by the vector

$$\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}.$$

We call $\vec{r}$ our points *position vector*.

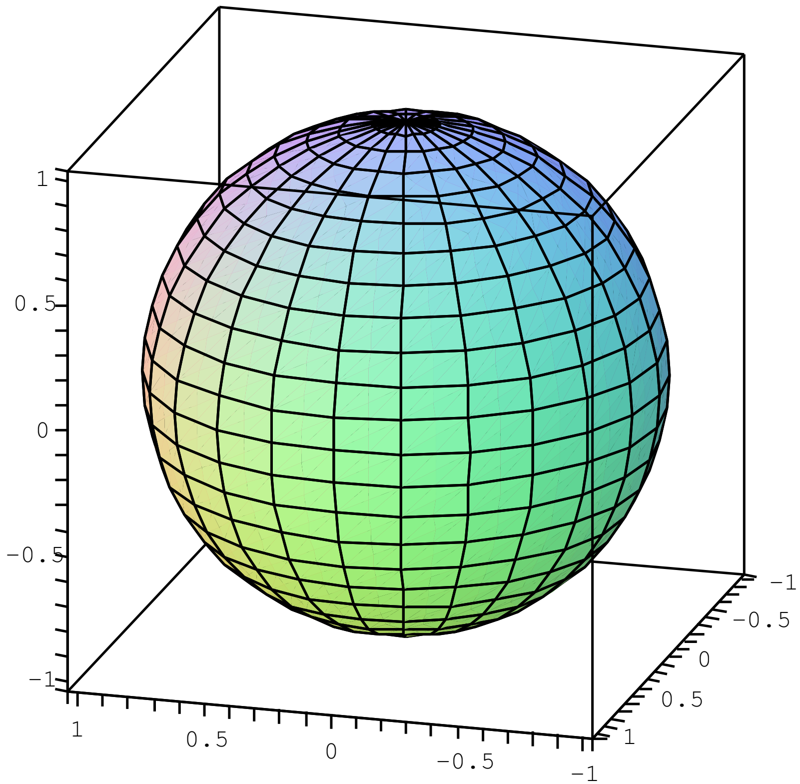
Ex: Describe the set points with position vectors satisfying $|\vec{r}| = 4$.

Displacement

In Euclidean space, staring at a point P with coordinate (x_1, y_1, z_1) and going to a point Q with coordinate (x_2, y_2, z_2) can be accomplished by via the *displacement vector*

$$\vec{PQ} = (x_2 - x_1)\hat{i} + (y_2 - y_1)\hat{j} + (z_2 - z_1)\hat{k}.$$

We are really lucky to do so!!!



Vector Addition

This uses the following notion of addition: Letting $\vec{v} =< x_1, y_1, z_1 >$ and $\vec{w} =< x_2, y_2, z_2 >$ we let

$$\begin{aligned} \vec{v} + \vec{w} &= < x_1, y_1, z_1 > + < x_2, y_2, z_2 > \\ &= < x_1 + x_2, y_1 + y_2, z_1 + z_2 > . \end{aligned}$$

Ex: Describe the set points with position vectors satisfying $|\vec{r} - < 1, 2, 0 > | = 4$.

Properties

Scalar multiplication and vector addition satisfy some rules that can be useful in manipulating them. Let $\vec{t}$, $\vec{v}$ and $\vec{w}$ be vectors and c be a scalar.

$$\vec{v}+\vec{w}=\vec{w}+\vec{v} \qquad commutativity$$

$$\vec{t}+(\vec{v}+\vec{w})=(\vec{t}+\vec{v})+\vec{w} \qquad Associativity$$

$$c(\vec{v}+\vec{w})=c\vec{v}+c\vec{w} \qquad Distributivity$$

Example

Let $\vec{a} =< -1, 2, 5 >$ and $\vec{b} =< 2, 2, 7 >$.

Find $|\vec{a}|$, $\vec{a} + \vec{b}$, $\vec{a} - \vec{b}$, and $3\vec{a} + 4\vec{b}$. Find the equation of a sphere centered at $\vec{a}$ that contains $\vec{b}$ in its boundary.