## LECTURE OUTLINE The Practice Exam

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Math 8

Oct. 13, 2004



# Review For Midterm Deciding On The Technique

### **Basic Techniques**

**Basic Rules:** (For example) If  $\sum a_n$  converges to s, then  $\sum ca_n$  converges to cs.

The Basic Sequence Fact:  $\lim_{x\to\infty} f(x) = L$ , then  $\lim_{n\to\infty} f(n) = L$ . Facts: geometric series  $\sum_{n=1}^{\infty} r^n = \frac{r}{1-r}$  and p-test  $\sum_{n=1}^{\infty} \frac{1}{n^p}$  converges if p > 1 and diverges for  $p \le 1$ .

#### Summation Tests

**Divergence Test:**  $\sum a_n$  diverges if  $\lim a_n \neq 0$ .

Integral Test: If f(x) is continuos, positive and decreasing on  $[1,\infty]$  and  $\int_1^{\infty} f(x) dx$  converges [or diverges], then  $\sum f(n)$  converges [or diverges].

**Comparison Test:** Suppose  $0 \le a_n \le b_n$ . If  $\sum b_n$  converges, then  $\sum a_n$  converges; while if  $\sum a_n$  diverges then  $\sum b_n$  diverges.

Limit Comparison Test: Suppose  $a_n$  and  $b_n$  are positive and  $0 < \lim_{n\to\infty} \frac{a_n}{b_n} = c < \infty$ . If  $\sum a_n$  converges [or diverges], then  $\sum b_n$  also converges [or diverges]. Alternating Series Test: Suppose  $b_n$  are positive with  $b_{n+1} < b_n$  and  $\lim_{n\to\infty} b_n = 0$ , then  $\sum (-1)^{n-1} b_n$  converges.

### **Integration Techniques**

When a a simple u-substitution does not seem to help.... **Integration by Parts:** When you see a product of distinct types of functions or you are integrating something whose derivative you know to be a rational function.

Trigonometric Substitution: When you see a  $\sqrt{a^2\pm x^2}$  type term .

**Trigonometric Integrals:** When you see powers of trig functions.

**Partial Fractions** When you are integrating a rational function (with degree of the numerator less than the degree of the denominator).