LECTURE OUTLINE Ratio Test

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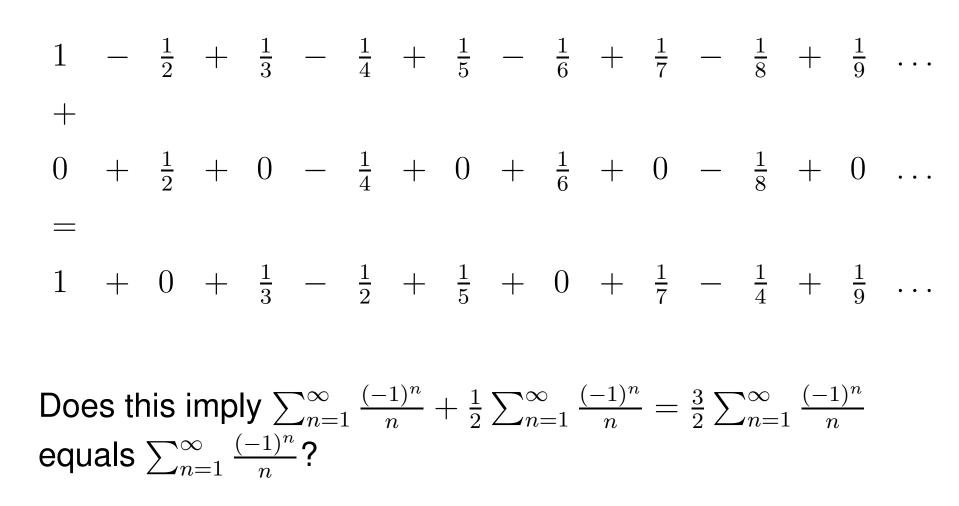
Math 8

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Rearrangement Theorem Ratio Test Root Test

Verify



Euler's γ *Constant*

To see the problem let us show

$$\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n} = \ln(2).$$

To do this, it is useful to think about the Euler's γ constant

$$\lim_{N \to \infty} \left(\sum_{n=1}^{N} \frac{1}{n} - \ln(N) \right) = \gamma \approx 0.57721566$$

Order Maters

By a *rearrangement* of an infinite series we mean a series obtained by simply changing the order of the terms.

Rearrangement Theorem:

If $\sum a_n$ is conditionally convergent and r is any real number, then there is a rearrangement of $\sum a_n$ such that $\sum a_n = r$.

If $\sum a_n$ is absolutely convergent with sum *s*, then any rearrangement of the $\sum a_n$ is convergent and has the same sum *s*.

Ratio Test

(1) If $\lim_{n\to\infty} \left| \frac{a_{n+1}}{a_n} \right| = L < 1$, then the series $\sum a_n$ is absolutely convergent.

(2) If $\lim_{n\to\infty} \left| \frac{a_{n+1}}{a_n} \right| = L > 1$ (or ∞), then the series $\sum a_n$ is divergent.

(3) If $\lim_{n\to\infty} \left| \frac{a_{n+1}}{a_n} \right| = 1$, then this test is inconclusive.

Ex: Is $\sum_{n=1}^{\infty} \frac{(-1)^{n-1}(n^3+n-2)}{\sqrt{3^n}}$ convergent? Ex: Is $\sum_{n=1}^{\infty} \frac{n^n}{n!}$ convergent?

Root Test

(1) If $\lim_{n\to\infty} (|a_n|)^{1/n} = L < 1$, then the series $\sum a_n$ is absolutely convergent.

(2) If $\lim_{n\to\infty} (|a_n|)^{1/n} = L > 1$ (or ∞), then the series $\sum a_n$ is divergent.

(3) If $\lim_{n\to\infty} (|a_n|)^{1/n} = 1$, then this test is inconclusive.

Ex: Is $\sum_{n=1}^{\infty} \left(\frac{3n+4}{7n-1}\right)^n$ convergent?