LECTURE OUTLINE
Ratio Test

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Math 8

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Goals

Rearrangement Theorem

Ratio Test

Root Test

Verify

$$1 \quad - \quad \frac{1}{2} \quad + \quad \frac{1}{3} \quad - \quad \frac{1}{4} \quad + \quad \frac{1}{5} \quad - \quad \frac{1}{6} \quad + \quad \frac{1}{7} \quad - \quad \frac{1}{8} \quad + \quad \frac{1}{9} \quad \ldots$$

$$+$$

$$0 \quad + \quad \frac{1}{2} \quad + \quad 0 \quad - \quad \frac{1}{4} \quad + \quad 0 \quad + \quad \frac{1}{6} \quad + \quad 0 \quad - \quad \frac{1}{8} \quad + \quad 0 \quad \ldots$$

$$=$$

$$1 \quad + \quad 0 \quad + \quad \frac{1}{3} \quad - \quad \frac{1}{2} \quad + \quad \frac{1}{5} \quad + \quad 0 \quad + \quad \frac{1}{7} \quad - \quad \frac{1}{4} \quad + \quad \frac{1}{9} \quad \ldots$$

Does this imply $\sum_{n=1}^{\infty} \frac{(-1)^n}{n} + \frac{1}{2} \sum_{n=1}^{\infty} \frac{(-1)^n}{n} = \frac{3}{2} \sum_{n=1}^{\infty} \frac{(-1)^n}{n}$ equals $\sum_{n=1}^{\infty} \frac{(-1)^n}{n}$?

Euler's γ Constant

To see the problem let us show

$$\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n} = \ln(2).$$

To do this, it is useful to think about the Euler's γ constant

$$\lim_{N\to\infty} \left(\sum_{n=1}^{N} \frac{1}{n} - \ln(N) \right) = \gamma \approx 0.57721566$$

Order Maters

By a *rearrangement* of an infinite series we mean a series obtained by simply changing the order of the terms.

Rearrangement Theorem:

If $\sum a_n$ is **conditionally convergent** and r is any real number, then there is a rearrangement of $\sum a_n$ such that $\sum a_n = r$.

If $\sum a_n$ is **absolutely convergent** with sum s, then any rearrangement of the $\sum a_n$ is convergent and has the same sum s.

Ratio Test

(1) If $\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| = L < 1$, then the series $\sum a_n$ is absolutely convergent.

(2) If $\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| = L > 1$ (or ∞), then the series $\sum a_n$ is divergent.

(3) If $\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| = 1$, then this test is inconclusive.

Ex: Is $\sum_{n=1}^{\infty} \frac{(-1)^{n-1}(n^3+n-2)}{\sqrt{3^n}}$ convergent?

Ex: Is $\sum_{n=1}^{\infty} \frac{n^n}{n!}$ convergent?

Root Test

(1) If $\lim_{n\to\infty}(|a_n|)^{1/n} = L < 1$, then the series $\sum a_n$ is absolutely convergent.

(2) If $\lim_{n\to\infty}(|a_n|)^{1/n} = L > 1$ (or ∞), then the series $\sum a_n$ is divergent.

(3) If $\lim_{n\to\infty}(|a_n|)^{1/n} = 1$, then this test is inconclusive.

Ex: Is $\sum_{n=1}^{\infty}\left(\frac{3n+4}{7n-1}\right)^n$ convergent?