# *LECTURE OUTLINE*
# *Multivariable Functions*

Professor Leibon

Math 8

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# *Goals*

Functions of Two Variable

Graphs

Contour maps

$$\frac{\partial f}{\partial x}(x, y)$$

# *Functions of Two Variable*

A *function of two variables* is a rule that assigns to each ordered pair of real numbers in a set $D$ (it's *domain*) a real number $f(x, y)$ (in its *range* $\{(f(x, y) \mid (x, y) \in D\}$).

**Example:** Let

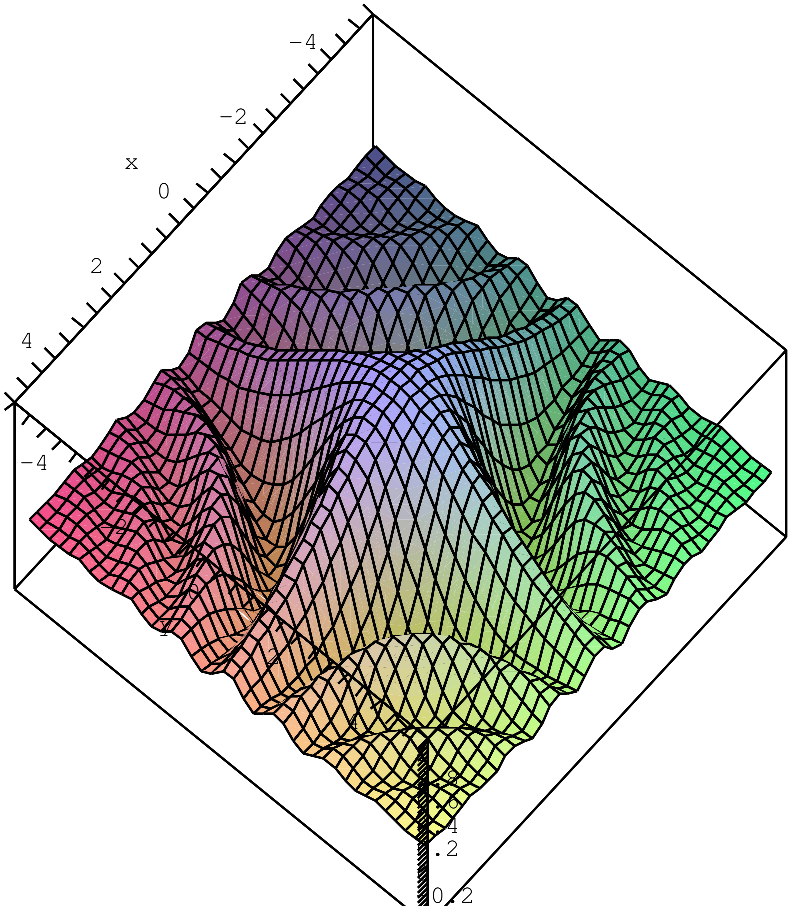
$$f(x, y) = \cos(xy)e^{\frac{-x^2-y^2}{10}}$$

with domain the whole plane. Compute $f(1, 0)$.

# *Mountain Range, graph*

The *graph* of a function of two variables with domain $D$ (it's *Domian*) is the set of points $(x, y, z)$ in $\mathbf{R}^3$ with $z = f(x, y)$ and $(x, y) \in D$.

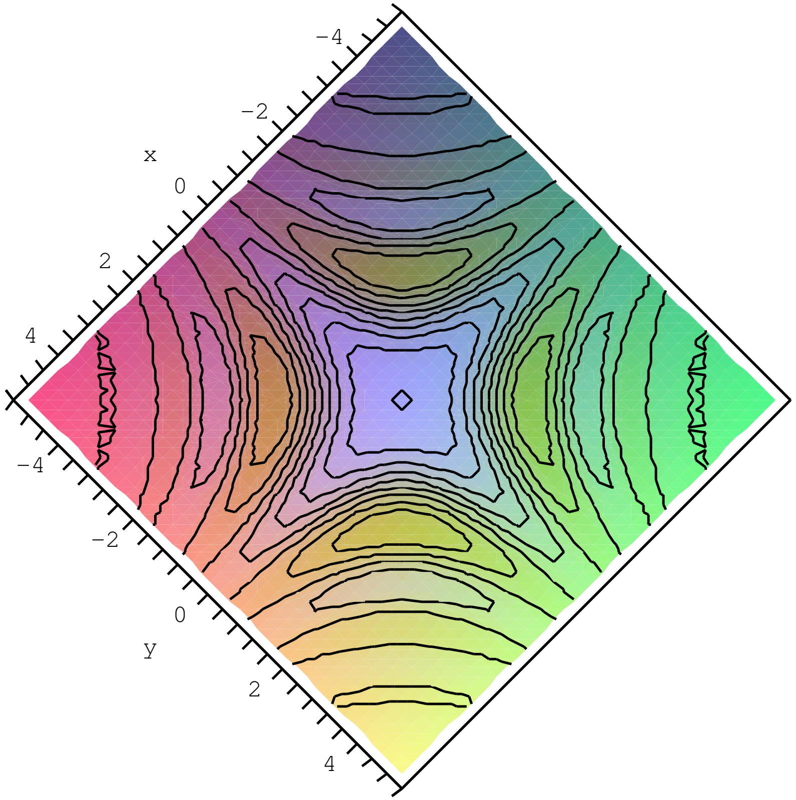
**Example:** $f(x, y) = \cos(xy)e^{\frac{-x^2-y^2}{10}}$ with domain $-5 \leq x \leq 5$ and $-5 \leq y \leq 5$.



# *Contour Plot topo map, level curves*

The *level curves* of a function of two variables are the curves with the equation $f(x,y,) = k$ where $k$ is constant (in $f$'s range).

**Example:** $f(x,y) = \cos(xy)e^{\frac{-x^2-y^2}{10}}$ with domain $-5 \le x \le 5$ and $-5 \le y \le 5$.



## *Example*

Determine a good domain, sketch the graph and sketch the level sets of

$$f(x, y) = \sqrt{16 - (x^2 + y^2)}$$

# *Limits*

A function of two variables is called continuous at $(a, b)$ if

$$lim_{(x,y)\to(a,b)} f(x, y) = f(a, b).$$

We say $f$ is continuous in $D$ if it is continuous at each point of $D$.

**Example:** Where are the following functions continuous?

$$f(x, y) = \frac{x^2 y}{x^2 + y^2}$$

$$f(x, y) = \frac{xy}{x^2 + y^2}$$

# *Partial Derivatives*

$\frac{\partial f}{\partial x}(x, y)$ means take the derivative in $x$ viewing $y$ as constant, in other words,

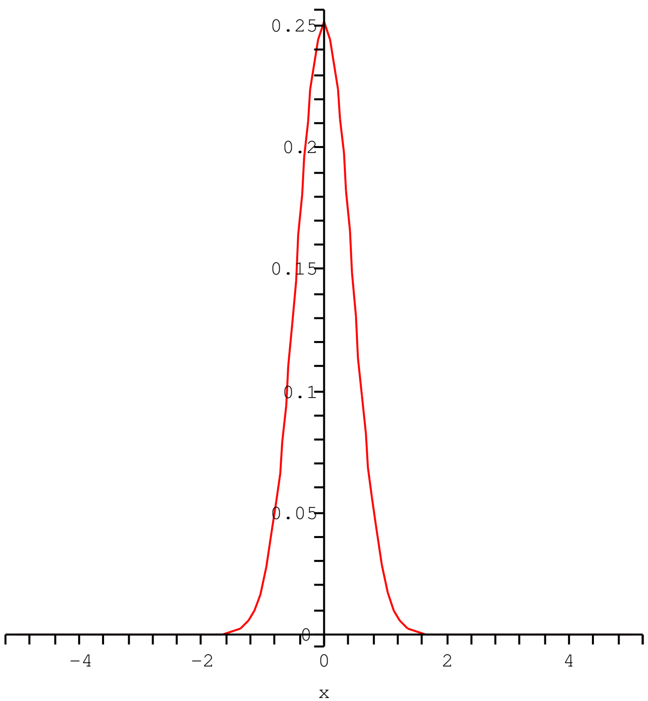
$$\frac{\partial f}{\partial x}(x, y) = f_x(x, y) = \lim_{h \to 0} \frac{f(x + h, y) - f(x, y)}{h}.$$

**Ex:** Find $\frac{\partial f}{\partial x}(x, y)$ where $f(x, y) = \cos(xy)e^{\frac{-x^2-y^2}{10}}$

# *Time*

We can also view a variable as *indexing* a family of functions in the other variable (often time).

$$f(x,t) = \frac{e^{-\frac{x^2}{4t}}}{\sqrt{4\pi t}}$$



## *Time*

$$f(x,t) = \frac{e^{-\frac{x^2}{4t}}}{\sqrt{4\pi t}}$$

**Ex:** Confirm $\frac{\partial f}{\partial t} = \frac{\partial^2 f}{\partial x^2}$.

# *Time*

$$f(x,t) = \int_{-\infty}^{\infty} \frac{e^{-\frac{(x-y)^2}{4t}}}{\sqrt{4\pi t}} g(y) dy$$

**Ex:** Explore the fact that $\frac{\partial f}{\partial t} = \frac{\partial^2 f}{\partial x^2}$, and $f(x,0) = g(y)$.