LECTURE OUTLINE

Lines

Professor Leibon

Math 8

Nov. 2, 2004

Goals

Parallelepiped Volume

Lines

Review

Given vectors $\vec{a}$ and $\vec{b}$ we define $\vec{a} \times \vec{b}$ to be the unique vector satisfying
(1) $\vec{a} \times \vec{b}$ is orthogonal to $\vec{a}$ and to $\vec{b}$ (or zero).
(2) It has length equal to the area of the parallelogram determined by $\vec{a}$ and $\vec{b}$.
(3) $\vec{a} \times \vec{b}$ is in the direction determined by the right hand rule going from $\vec{a}$ to $\vec{b}$.

Theorem:

$$(\vec{a} \times \vec{b}) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}.$$

Parallelepiped Volume

The *oriented volume* of the parallelepiped determined by $\vec{a}$, $\vec{b}$, and $\vec{c}$ is given by

$$\vec{a} \cdot (\vec{b} \times \vec{c}) \equiv \det \begin{pmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{pmatrix} \equiv \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}.$$

Let $\vec{a} =< -1, 2, 5 >$, $\vec{b} =< 2, 2, 7 >$ and $\vec{c} =< 3, 2, -1 >$.

Find the oriented volume of the parallelepiped determined by $\vec{a}$, $\vec{b}$. and $\vec{c}$. (The sign tells you whether the right hand rule has been respected.)

Lines

Given a point $\vec{r}_0 =< x_0, y_0, z_0 >$ and direction $\vec{v} =< a, b, c >$, we can form a *parameterized* line that starts at $\vec{r}_0$ and and travels in the direction $\hat{v}$ with speed $|\vec{v}|$, via

$$\vec{r}(t) = \vec{r}_0 + t\vec{v} =< x_0 + ta, y_0 + tb, z_0 + tc > .$$

We can implicitly describe this line by solving for t, we have

$$\frac{x - x_0}{a} = \frac{y - y_0}{b} = \frac{z - z_0}{c}.$$

Lines: Example 1

1. Find a parameterization and an implicit description of the line parallel to the vector $< 1, 1, -2 >$ and containing the point $(2, 3, 8)$.

2. Does this line ever pass through a point with y coordinate 9? If so find this point.

Lines: Example 2

1. Find a parameterization of the line containing the points $(1, 3, 2)$ and $(-3, 5, 7)$.

2. At what point does this line intersect the xy-plane?

Lines: Example 3

Find a parameterization of the line perpendicular to the directions $\vec{a} = < -1, 2, 5 >$ and $\vec{b} = < 2, 2, 7 >$ and going through the point $(1, 2, 0)$.

Lines

Given a points $\vec{r}_0 =< x_0, y_0, z_0 >$ and $\vec{r}_1 =< x_1, y_1, z_1 >$ we can form a *parameterized* line that starts at $\vec{r}_0$ at time 0 and travels at constant speed to $\vec{r}_1 =< x_1, y_1, z_1 >$ arriving there at time 1 via

$$\vec{r}(t) = (1-t)\vec{r}_0 + t\vec{r}_1 =< (1-t)x_0 + tx_1, (1-t)y_0 + ty_1, (1-t)z_0 + tz_1 >$$

Find a parameterization of the line, that start at time zero at $(1, 3, 2)$ and ends at time 1 at $(-3, 5, 7)$.