LECTURE OUTLINE
Directional Derivative

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Math 8

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Goals

Directional Derivative

Level Surfaces

Review

Let $\nabla f =< \frac{\partial f}{\partial x_1}, \ldots, \frac{\partial f}{\partial x_n} >$ (we called this the *gradient*) and $\vec{x} =< x_1, \ldots, x_n >$ with each x_i a function of the variables $t_1 \ldots t_m$, then we have the *chain rule*

$$\frac{\partial f}{\partial t_i} = \nabla f \cdot \frac{\partial \vec{x}}{\partial t_i}.$$

Example: An elliptical balloon described by $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$ has volume given by $\frac{4\pi}{3} abc$. Suppose this balloon is being blown up so that when $(a, b, c) = (2, 4, 5)$ we have that $\frac{d}{dt}(a, b, c) = (.1, .2, .25)$. At what rate is the balloon's volume increasing when $(a, b, c) = (2, 4, 5)$?

Directional Derivative: Two dimensions

Given a direction $\hat{u} = a\hat{i} + b\hat{j}$, the *directional derivative* of $f(x, y)$ at (x_0, y_0) in the direction $\hat{u}$ is

$$\frac{df(x_0 + at, y_0 + bt)}{dt} = \nabla f(x_0, y_0) \cdot < a, b > .$$

Example: Suppose the gradient of f at $(2, 3)$ is $< -1, 5 >$. At what rate does f change as one heads in the direction $\frac{1}{\sqrt{2}}\hat{i} + \frac{1}{\sqrt{2}}\hat{j}$ starting at $(2, 3)$?

The Directional Derivative Function

Given a direction $\hat{u} = a\hat{i} + b\hat{j}$ the *directional derivative function* associated to $f(x, y)$ is

$$D_u f(x, y) = \frac{df(x + at, y + bt)}{dt} = \nabla f(x, y) \cdot < a, b > .$$

Example: Find the directional derivative function associated to $f(x, y) = ye^{x^2}$ in the direction $\frac{1}{2}\hat{i} - \frac{\sqrt{3}}{2}\hat{j}$.

Directional Derivative: Higher Dimensions

Given a direction $\hat{u}$ the *directional derivative function* associated to f in the direction $\hat{u}$ is

$$D_u f = \nabla f \cdot \vec{u}.$$

Example: Find the directional derivative function associated to $f(x, y, z) = x^2 + y^2 + z^2$ in the direction $\hat{k}$.

Using the dot product

Let θ be the angle between $\hat{u}$ and ∇f. Notice, we have

$$D_u f = |\nabla f| \cos(\theta)$$

Consequences: f increases the fastest in the direction of ∇f, f decreases fastest the direction of $-\nabla f$, and f does not change as we head in a direction perpendicular to ∇f.

Contour...