LECTURE OUTLINE

Tangent Planes

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Math 8

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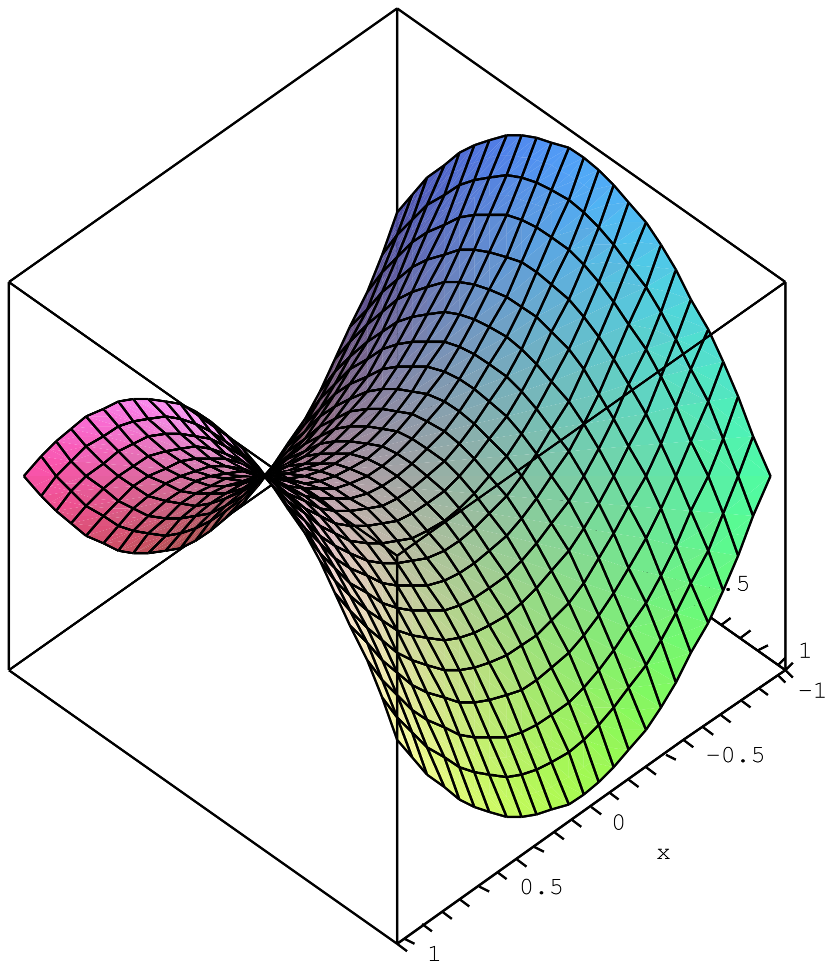
Goals

Tangent Planes

Linear Approximation

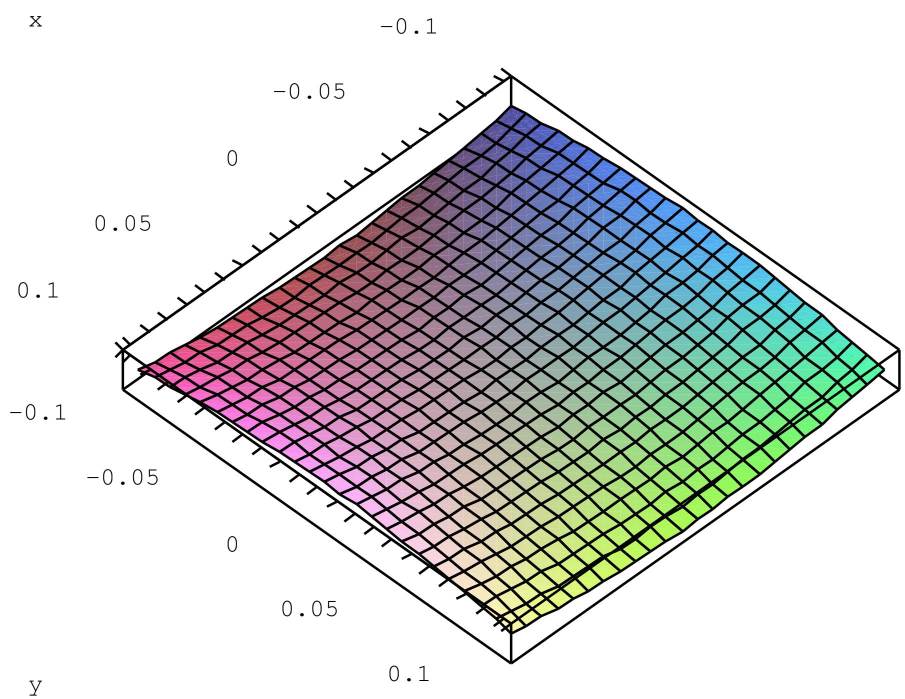
Tangent Planes

Example: $f(x, y) = x^2 - y^2$ at $(0, 0, 0)$.



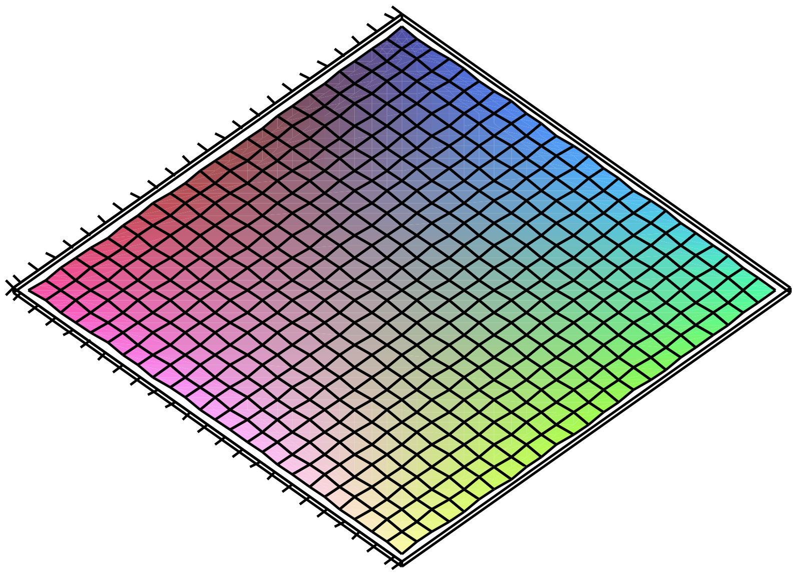
Tangent Plane

Example: $f(x, y) = x^2 - y^2$ at $(0, 0, 0)$. Zoom in towards $(0, 0, 0)$....



Tangent Plane

Example: $f(x, y) = x^2 - y^2$ at $(0, 0, 0)$. Zoom in towards $(0, 0, 0)$ and we see a plane, the tangent plane.



Tangent Plane

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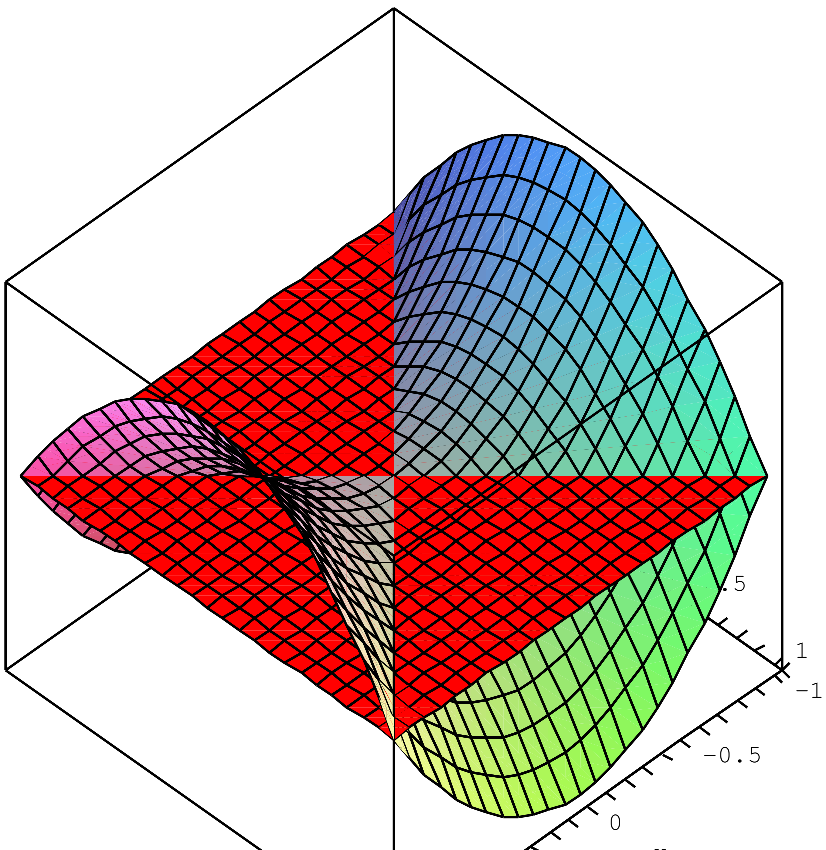
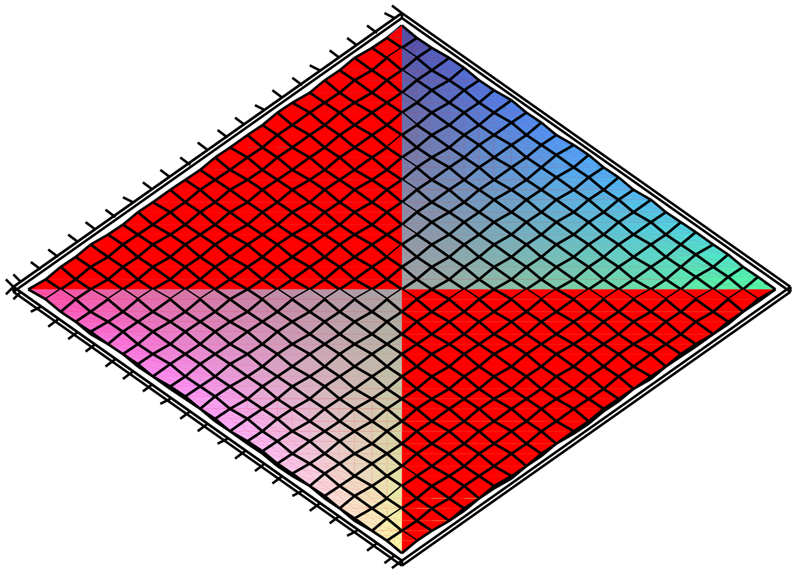
Example: $f(x, y) = x^2 - y^2$ near $(0, 0, 0)$.



Tangent Plane

Example: $f(x, y) = x^2 - y^2$ near $(0, 0, 0)$.



Tangent Plane

Provided a tangent plane exist, to find it we need two vectors. Argue that $< 1, 0, \frac{\partial f}{\partial x}(a, b) >$ and $< 0, 1, \frac{\partial f}{\partial y}(a, b) >$ should be in the tangent plane at $(a, b, f(a, b))$. From this the plane's normal is

$$\vec{n} =< -\frac{\partial f}{\partial x}(a, b) - \frac{\partial f}{\partial y}(a, b), 1 > .$$

Example: Find the tangent plane of $f(x, y) = x^2 - y^2$ at $(2, 1, 3)$.

Linear Approximation

We can approximate our function with this plane, namely

$$f(x,y) \approx f(a,b) + \frac{\partial f}{\partial x}(a,b)(x-a) + \frac{\partial f}{\partial y}(a,b)(y-b).$$

Example: Approximate the value of $f(x,y) = x^2 - y^2$ at $(2.05, 1.03)$. How close is it the true value?

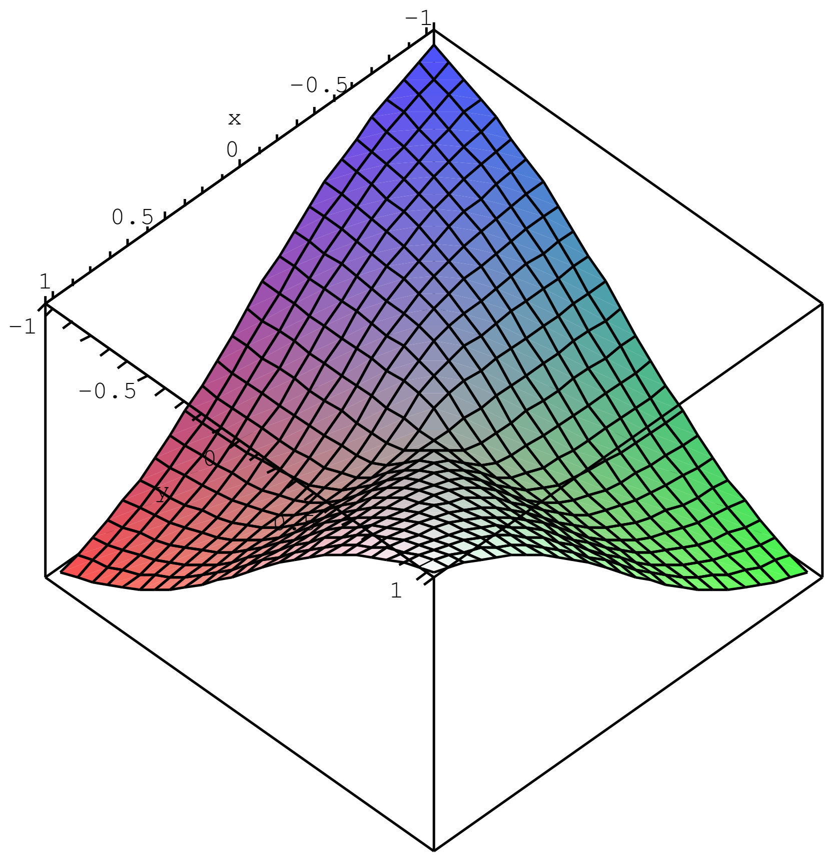
A Cruel and UNUSUAL example

The tangent plane may fail to exist if $\frac{\partial f}{\partial x}$, $\frac{\partial f}{\partial y}$ are **not** continuous. In other words, be careful when a denominator takes on a zero, or when function can't make up its mind about a certain value.

Ex. Let $f(x,y) = \frac{xy}{\sqrt{x^2+y^2}}$. Find f_x and f_y. Are they continuos?

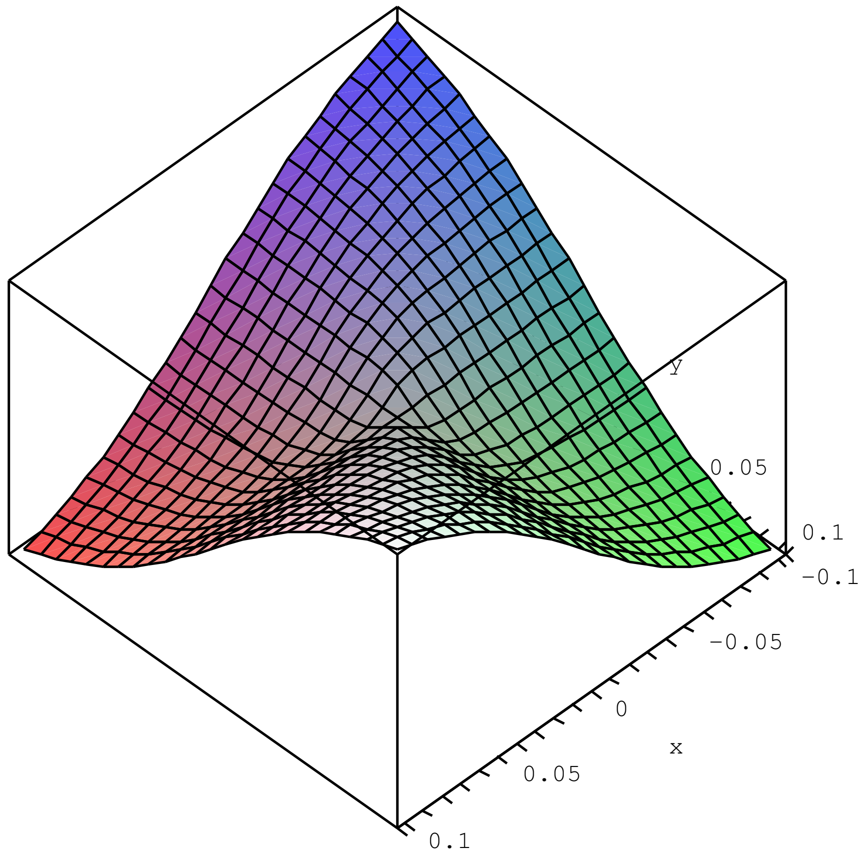
The Non-Tangent Plane

Let $f(x,y) = \frac{xy}{\sqrt{x^2+y^2}}$. Zoom in towards the $(0,0,0)$...



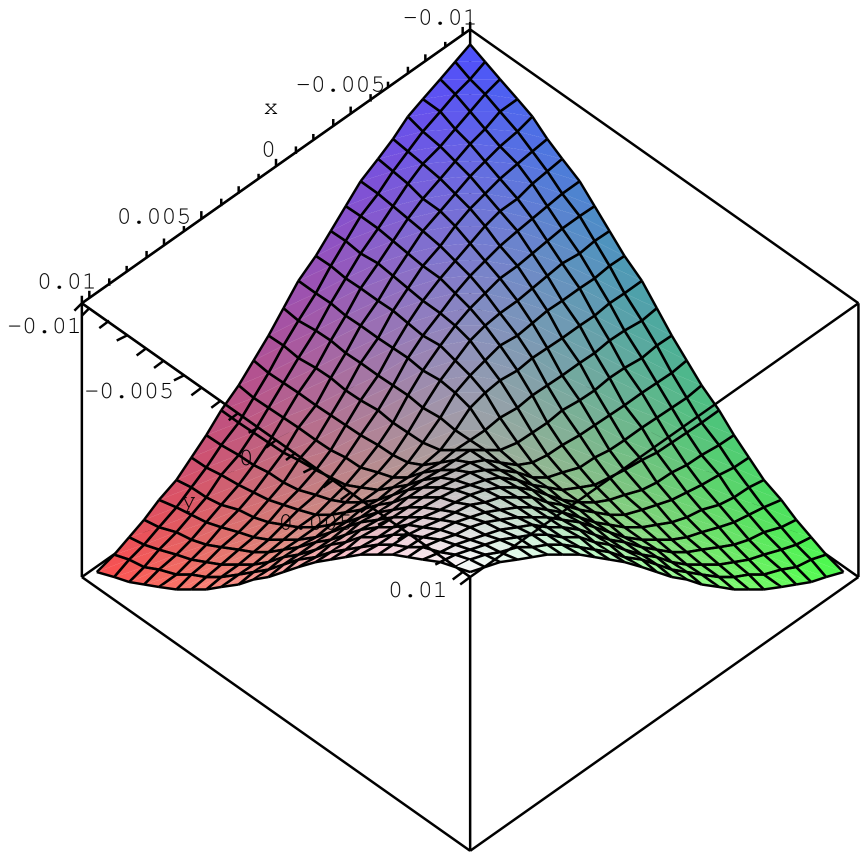
The Non-Tangent Plane

Let $f(x, y) = \frac{xy}{\sqrt{x^2+y^2}}$. Zoom in towards the $(0, 0, 0)$, and,...



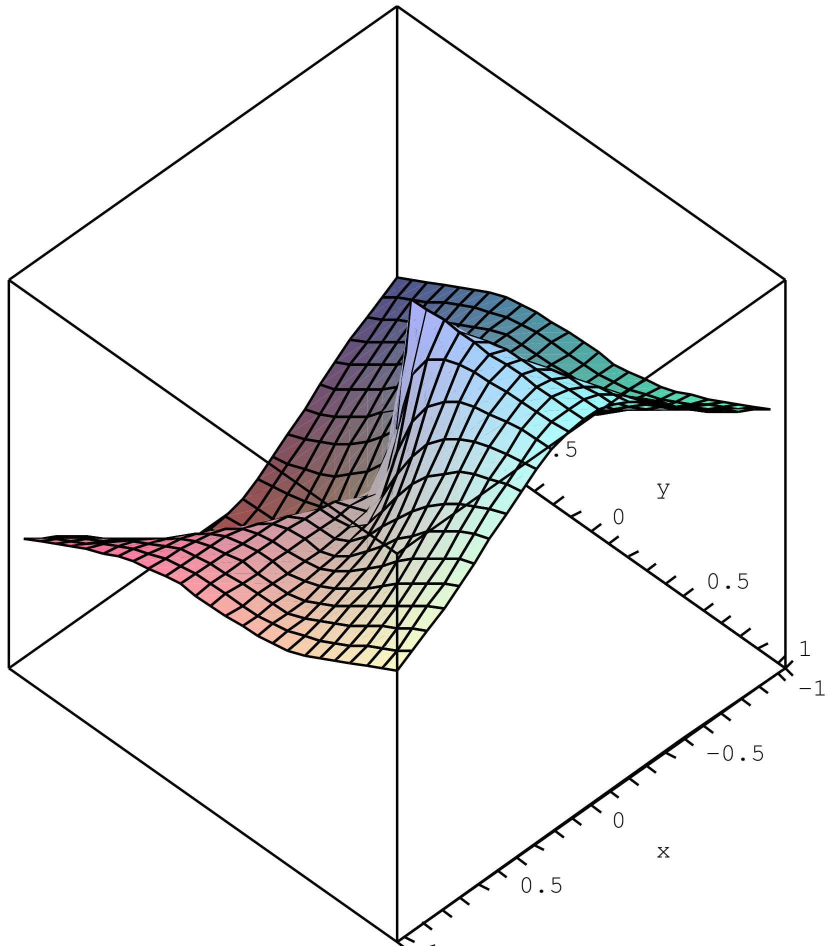
The Non-Tangent Plane

Let $f(x, y) = \frac{xy}{\sqrt{x^2+y^2}}$. Zoom in towards the $(0, 0, 0)$, and nothing happens!



The Non-Tangent Plane

Let $f(x, y) = \frac{xy}{\sqrt{x^2+y^2}}$ and zoom in towards $(0, 0, 0)$ on the graph of $\frac{\partial f}{\partial x}$...



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The Non-Tangent Plane

Let $f(x,y) = \frac{xy}{\sqrt{x^2+y^2}}$ and zoom in towards $(0,0,0)$ on the graph of $\frac{\partial f}{\partial x}$ and EEEEEKKKK!!!!!

The Differential

We can approximate our function with this plane. As such, near (a, b) we have $\Delta z = f(x, y) - f(a, b)$ is approximately

$$\frac{\partial f}{\partial x}(a, b)(x - a) + \frac{\partial f}{\partial y}(a, b)(y - b) = \frac{\partial f}{\partial x}\Delta x + \frac{\partial f}{\partial y}\Delta y.$$

It can be useful to think using the *differential*

$$dz = \frac{\partial f}{\partial x}dx + \frac{\partial f}{\partial y}dy.$$

Example: Find the differential of $f(x, y) = x^2 - y^2$ at $(2, 1, 3)$.

Limits

A function of two variables is called continuous at (a, b) if

$$lim_{(x,y)\to(a,b)}f(x, y) = f(a, b).$$

We say f is continuous in D if it is continuous at each point of D.

Example: Show $f(x, y) = \frac{xy}{\sqrt{x^2+y^2}}$ is continuos at zero, but that f_x is not.