LECTURE OUTLINE
Partial Derivatives

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Math 8

Nov. 12, 2004

Goals

Partial Derivatives

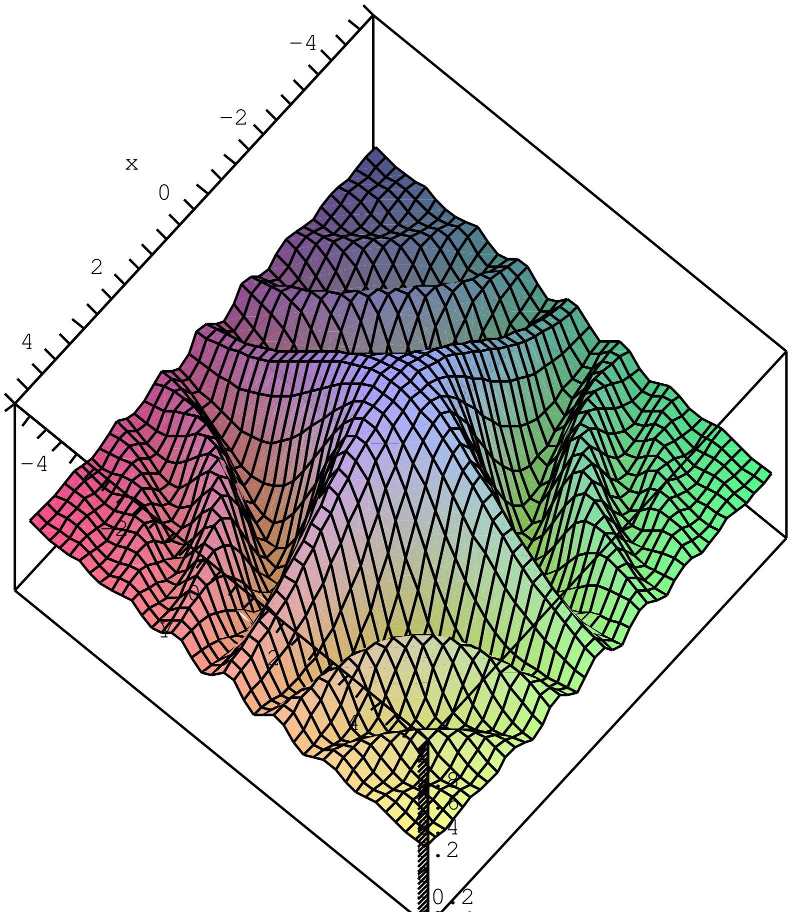
$$\frac{\partial f}{\partial x}(x, y)$$

Partial Differential Equations

Tangent Planes

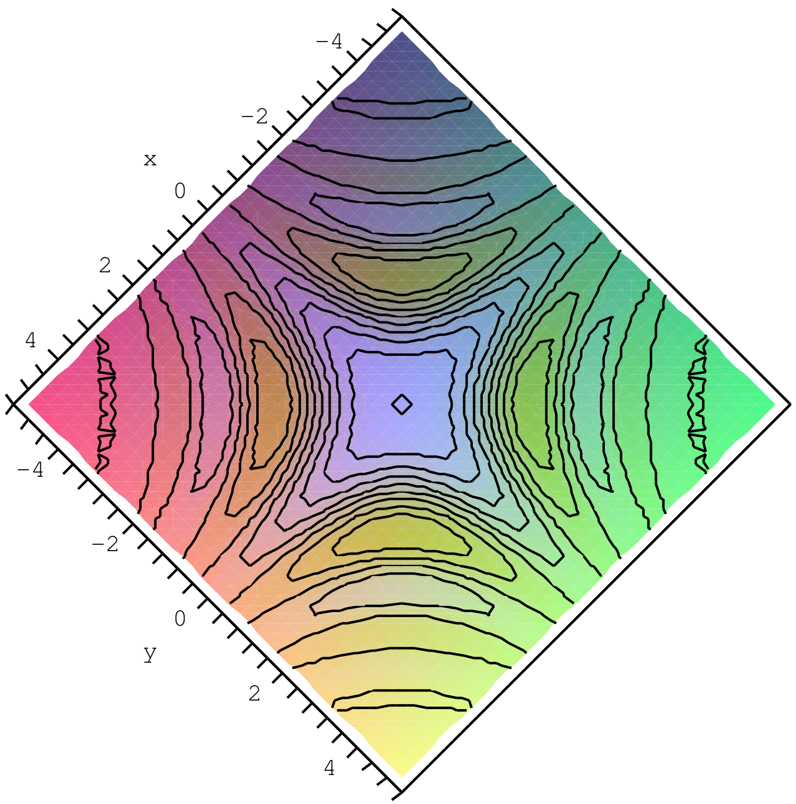
Review: Graph

Example: $f(x, y) = \cos(xy)e^{\frac{-x^2-y^2}{10}}$ with domain $-5 \le x \le 5$ and $-5 \le y \le 5$.



Review: Contour Plot

Example: $f(x,y) = \cos(xy)e^{\frac{-x^2-y^2}{10}}$ with domain $-5 \le x \le 5$ and $-5 \le y \le 5$.



Partial Derivatives

$\frac{\partial f}{\partial x}(x, y)$ means take the derivative in x viewing y as constant, in other words,

$$\frac{\partial f}{\partial x}(x, y) = f_x(x, y) = \lim_{h \to 0} \frac{f(x + h, y) - f(x, y)}{h}.$$

Ex: Find f_x and f_y when $\frac{xy}{\sqrt{x^2+y^2}}$.

Higher Derivatives

Let $f(x, y) = x^3 - y^3$. Find f_{xx} and f_{yy}.

A *partial differential equation* (PDE) is an equation like this:

$$f_{xx} + f_{yy} = 0$$

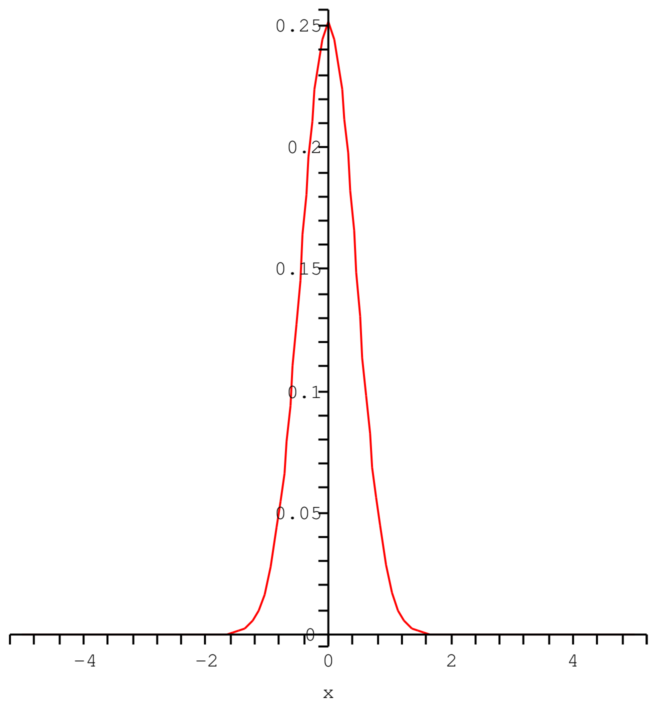
This is called *Laplace's Equation*. We try and find *solutions* to a PDE, namely functions that solve the given equation.

Find a solution to Laplace's equation.

Time

We can also view a variable as *indexing* a family of functions in the other variable (often time).

$$f(x,t) = \frac{e^{-\frac{x^2}{4t}}}{\sqrt{4\pi t}}$$



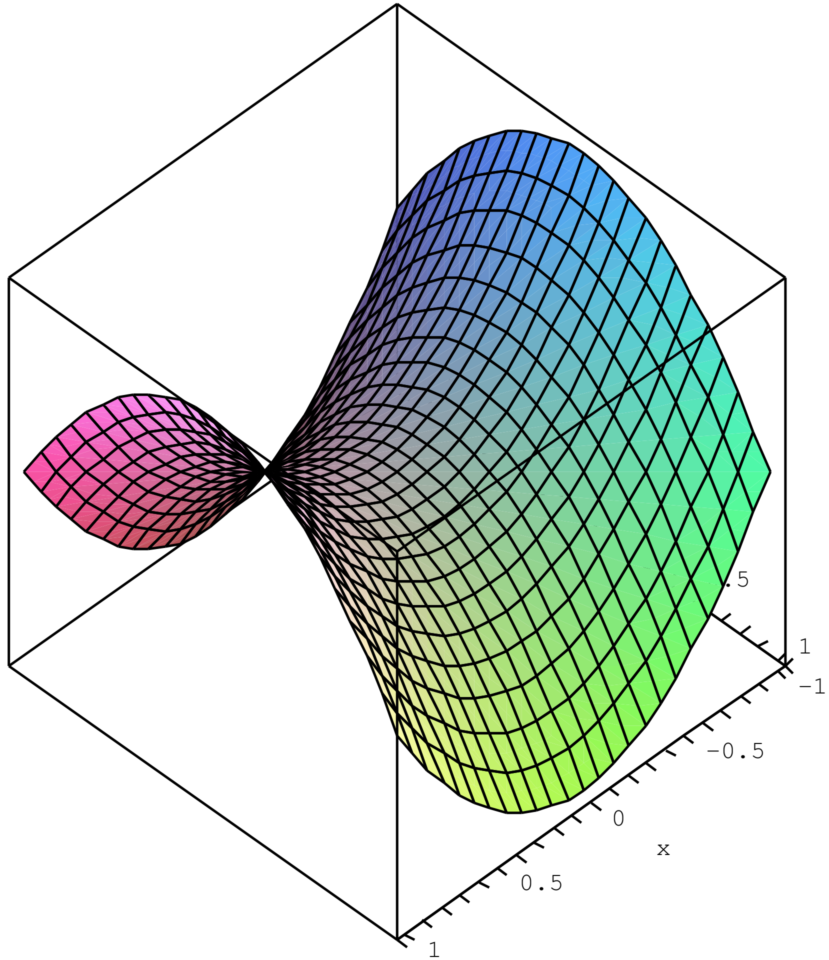
Another PDE

$$f(x,t) = \frac{e^{-\frac{x^2}{4t}}}{\sqrt{4\pi t}}$$

Ex: Confirm $f_t = f_{xx}$, the *heat equation*.

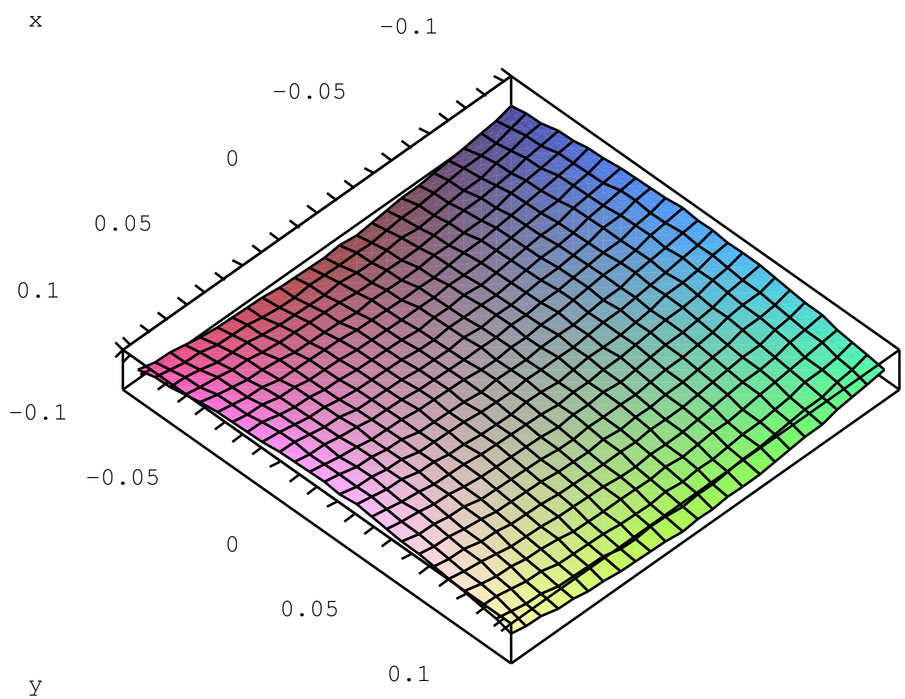
Tangent Planes

Example: $f(x, y) = x^2 - y^2$ at $(0, 0, 0)$.



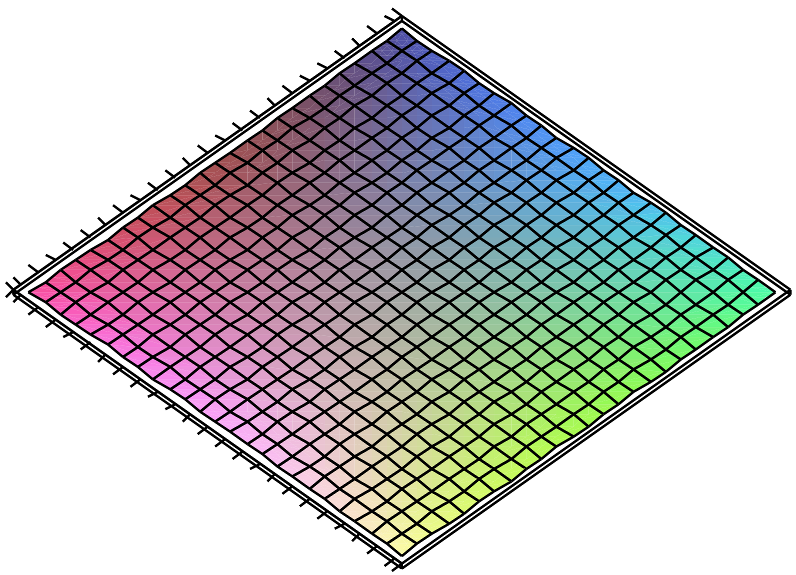
Tangent Plane

Example: $f(x, y) = x^2 - y^2$ at $(0, 0, 0)$. Zoom in towards $(0, 0, 0)$....



Tangent Plane

Example: $f(x, y) = x^2 - y^2$ at $(0, 0, 0)$. Zoom in towards $(0, 0, 0)$ and we see a plane, the tangent plane.

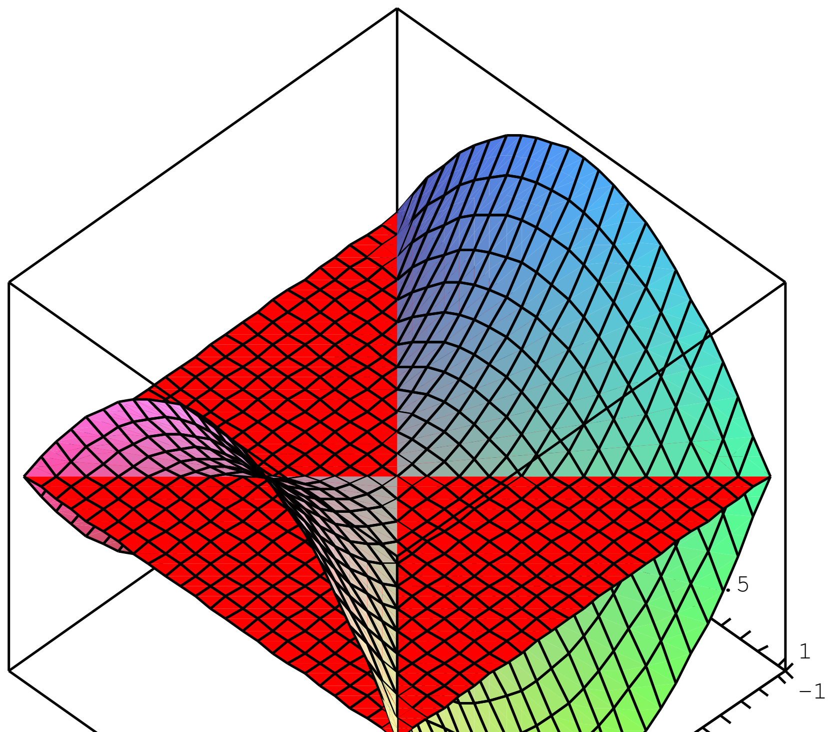


Tangent Plane

In other words: near (x_0, y_0) we have that $f(x, y)$ looks like

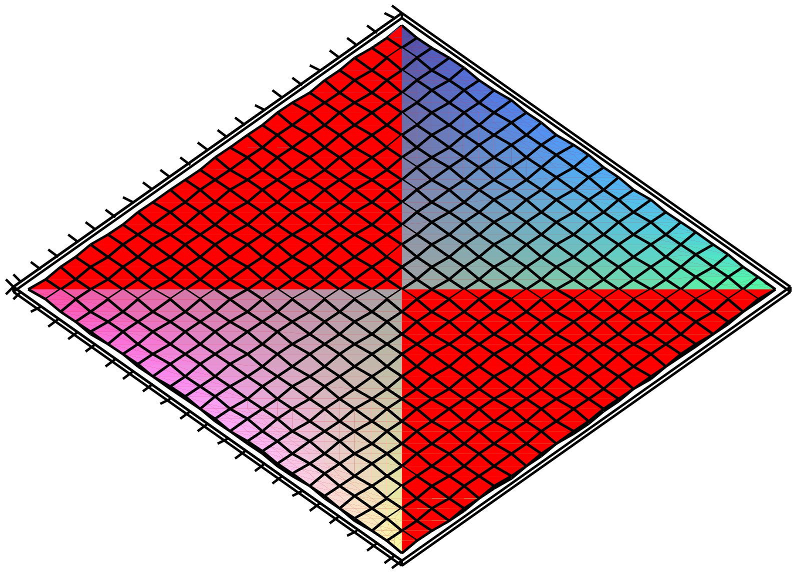
$$z = f(x_0, y_0) + \nabla f \cdot (x - x_0, y - y_0).$$

Example: $f(x, y) = x^2 - y^2$ near $(0, 0, 0)$.



Tangent Plane

Example: $f(x, y) = x^2 - y^2$ near $(0, 0, 0)$.



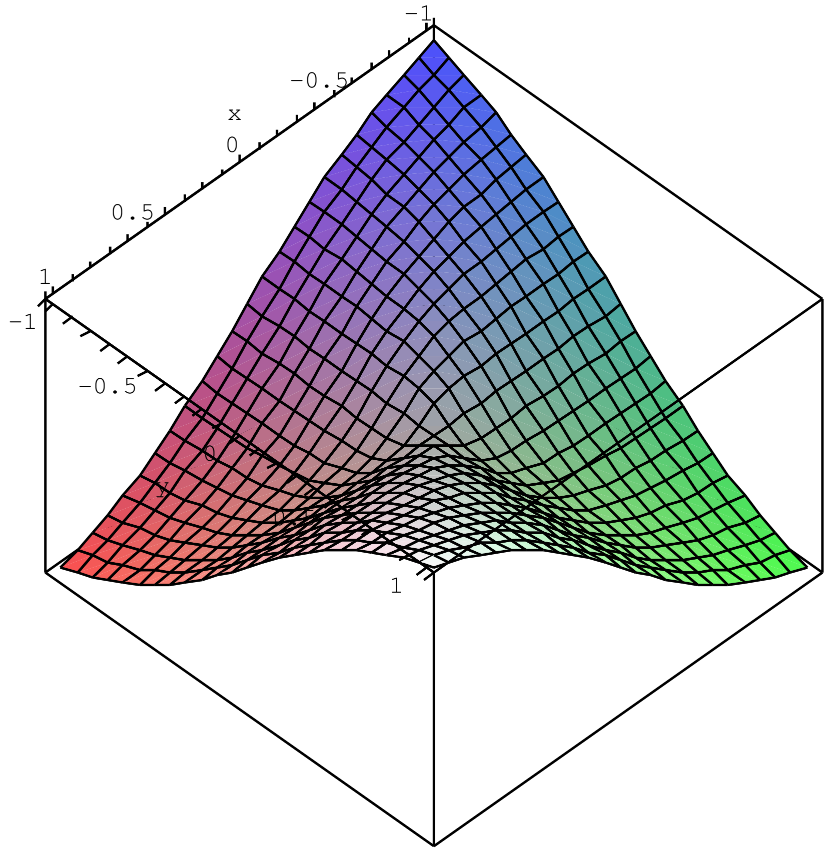
A Cruel and UNUSUAL example

The tangent plane may fail to exist if $\frac{\partial f}{\partial x}$, $\frac{\partial f}{\partial y}$ are **not** continuous. In other words, be careful when a denominator takes on a zero, or when function can't make up its mind about a certain value.

Ex. Let $f(x,y) = \frac{xy}{\sqrt{x^2+y^2}}$. Find f_x and f_y. Are they continuos?

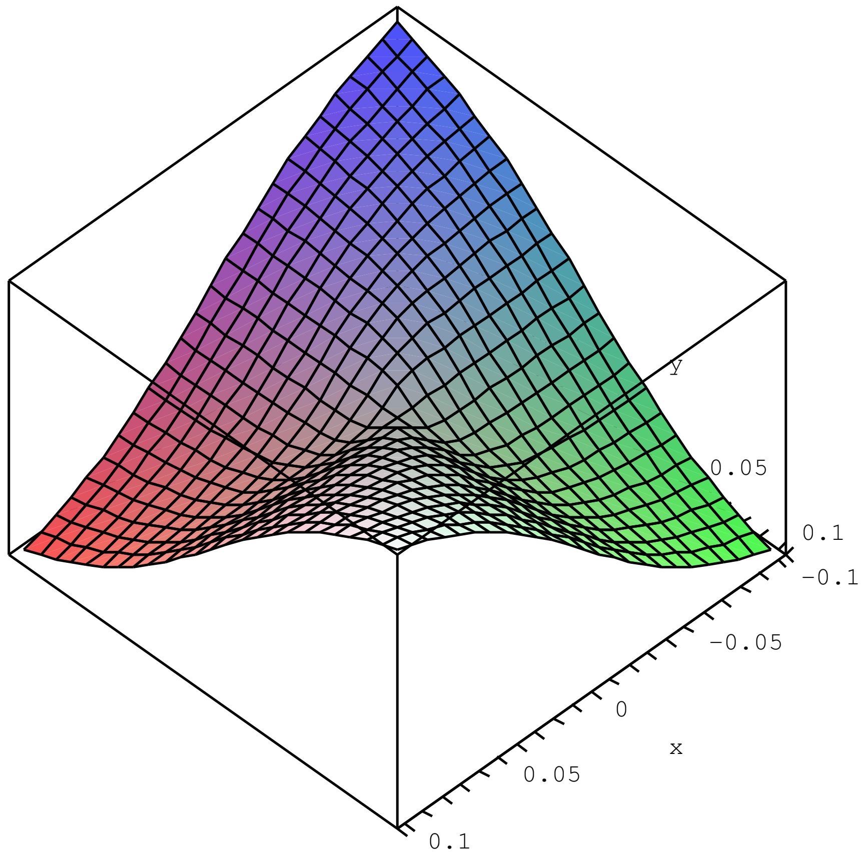
The Non-Tangent Plane

Let $f(x,y) = \frac{xy}{\sqrt{x^2+y^2}}$. Zoom in towards the $(0,0,0)$...



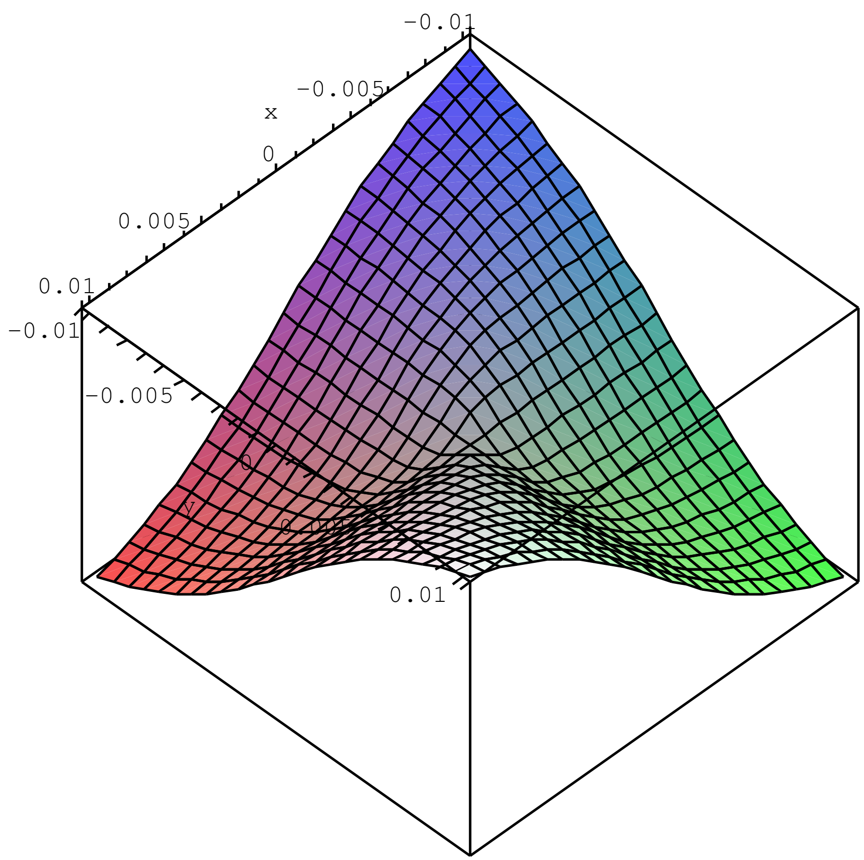
The Non-Tangent Plane

Let $f(x, y) = \frac{xy}{\sqrt{x^2+y^2}}$. Zoom in towards the $(0, 0, 0)$, and,...



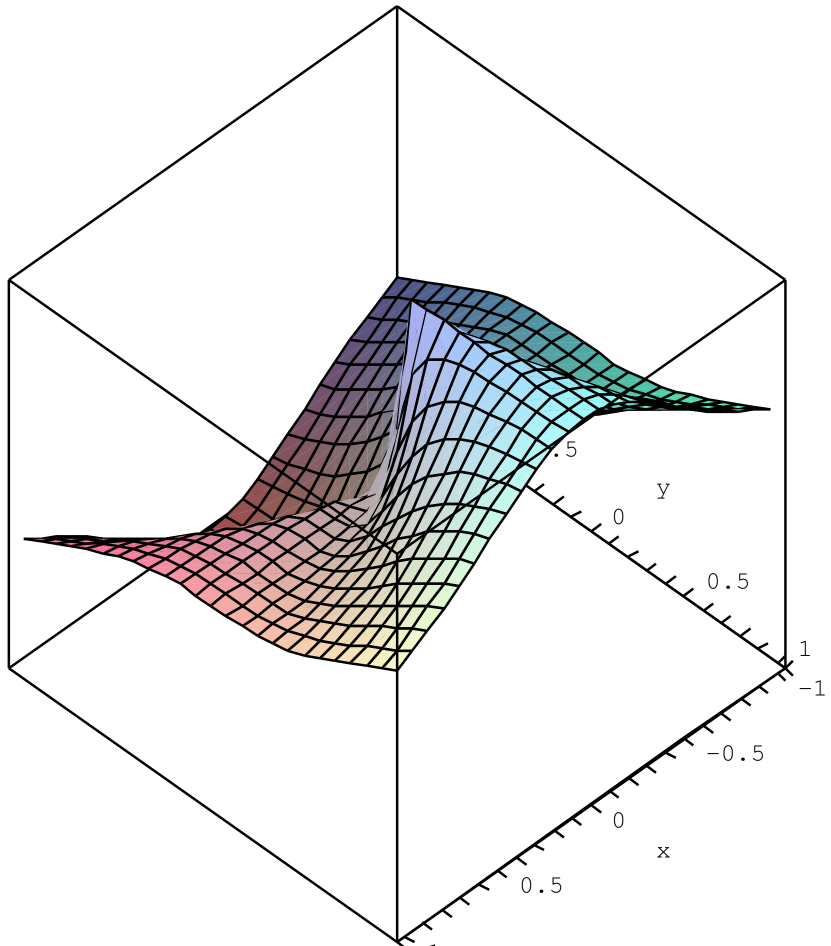
The Non-Tangent Plane

Let $f(x, y) = \frac{xy}{\sqrt{x^2+y^2}}$. Zoom in towards the $(0, 0, 0)$, and nothing happens!



The Non-Tangent Plane

Let $f(x,y) = \frac{xy}{\sqrt{x^2+y^2}}$ and zoom in towards $(0,0,0)$ on the graph of $\frac{\partial f}{\partial x}$...



The Non-Tangent Plane

Let $f(x,y) = \frac{xy}{\sqrt{x^2+y^2}}$ and zoom in towards $(0,0,0)$ on the graph of $\frac{\partial f}{\partial x}$...

The Non-Tangent Plane

Let $f(x, y) = \frac{xy}{\sqrt{x^2+y^2}}$ and zoom in towards $(0, 0, 0)$ on the graph of $\frac{\partial f}{\partial x}$ and EEEEEKKKK!!!!!

Limits

A function of two variables is called continuous at (a, b) if

$$lim_{(x,y)\to(a,b)} f(x, y) = f(a, b).$$

We say f is continuous in D if it is continuous at each point of D.

Example: Show $f(x, y) = \frac{xy}{\sqrt{x^2+y^2}}$ is continuos at zero, but that f_x is not.