1. (20) (**Show all work**).

   As a model rocket is launched, it is bumped slightly causing it to have an rather erratic flight path. It is observed that the acceleration of the rocket is given by $\mathbf{a}(t) = \langle -5\cos t, -2\sin t, 0\rangle$ m/sec^2, with an initial velocity of $\mathbf{v}(0) = \langle 0, 2, 6000/\pi\rangle$ m/sec, and an initial position of $\mathbf{r}(0) = \langle 5, 0, 0\rangle$. **Note:** There are no other forces acting on the rocket, so don't bring gravity into the picture.

   (a) Determine the velocity $\mathbf{v}(t)$ of the rocket at time t.

   (b) Determine the position $\mathbf{r}(t)$ of the rocket at time t.

   (c) How long does it take for the rocket to reach an altitude of 3000 m?

   (d) A balloon happens to be drifting by following the straight line path $\langle 0, 0, 3000\rangle + s\langle 1, 2, 0\rangle$. Does the balloon get hit by the rocket? Why or why not?

2. (20) (**Show all work**). Lines and planes

   (a) Find an equation of the line of intersection of the two planes $x + 2y + 3z = 2$ and $3x - 2y + z = 2$.

   (b) Find the point of intersection of the line $\mathbf{r} = \langle 1 + 2t, 3 + 4t, 5 + 6t\rangle$ and the plane $x + 2y + 3z = 50$.

3. (60) **Multiple Choice** Circle the correct response. (No partial credit will be given)

   (a) Consider the planes (1) $2(x - 2) + 3(y - 1) - 4(z + 1) = 0$,
   (2) $2x + 3y - 2z = 11$, and (3) $x + (3/2)y - 2z = 3$.

   A. (1) & (2) are parallel **B**. (2) & (3) are parallel

   C. (1) & (3) are parallel **D**. (1), (2) & (3) are parallel

   E. none of the above

   (b) Consider the cube determined by the vectors **i**, **j** and **k**. The cosine of the angle between the diagonal from $(0, 0, 0)$ to $(1, 1, 1)$ and the diagonal from $(1, 1, 0)$ to $(0, 0, 1)$ is

   A. $-1/\sqrt{3}$ **B**. $\pi/2$ **C**. $1/3$ **D**. $\sqrt{3}/3$

   E. none of the above

   (c) Let P be the plane given by $x + y + z = 3$ and L the line given by $x = t + 2$, $y = -t + 2$, $z = t + 2$. Then

A. L lies on P **B**. L and P are perpendicular

C. L and P are parallel, but L not on P

D. L and P intersect at one point **E**. none of the above

(d) If L_1 is the line given by $x = 3t + 1$, $y = 2t - 1$, $z = t + 3$ and the line L_2 is given by $x = 2s + 7$, $y = 2s + 3$, $z = s + 5$. Then L_1 and L_2 are

A. intersecting **B**. parallel, but not equal **C**. skew

D. equal **E**. none of the above

(e) If the vectors $\mathbf{a}$ and $\mathbf{b}$ determine adjacent sides of a parallelogram, then the two diagonals can be written as

A. $\mathbf{a} - \mathbf{b}$, $-\mathbf{a} - \mathbf{b}$ **B**. $\mathbf{a} + \mathbf{b}$, $\mathbf{a} - \mathbf{b}$ **C**. $\mathbf{a} - \mathbf{b}$, $\mathbf{b} - \mathbf{a}$

D. $\mathbf{a} + \mathbf{b}$, $-\mathbf{a}$ **E**. none of the above

(f) If $\mathbf{r}(t) = \langle t^2, \ln t, 2t \rangle$ then the length of $\mathbf{r}$ from $t = 1$ to $t = 2$ is

A. $2 + e$ **B**. $3 + \ln 2$ **C**. 3.6 **D**. 3

E. none of the above

(g) For a curve $\mathbf{u}(t)$, $\dfrac{d|\mathbf{u}|}{dt} =$

A. $\dfrac{\mathbf{u}'}{2\sqrt{|\mathbf{u}|}}$ **B**. $|\mathbf{u}'|$ **C**. $\dfrac{\mathbf{u}'}{2|\mathbf{u}|}$ **D**. $\dfrac{\mathbf{u} \bullet \mathbf{u}'}{|\mathbf{u}|}$

E. none of the above

(h) If the motion of a particle is given by a vector valued function $\mathbf{r}(t)$ defined for $a \leq t \leq b$, then the integral of the speed of $\mathbf{r}(t)$ from $t = a$ to $t = b$ equals

A. the acceleration **B.** the distance from $\mathbf{r}(a)$ to $\mathbf{r}(b)$

C. the velocity

D. the distance the particle travels from $\mathbf{r}(a)$ to $\mathbf{r}(b)$

E. none of the above

(i) Consider the vector-valued functions $\mathbf{r}(t) = \langle t, t, t \rangle, 1 \leq t \leq e$ and, $\mathbf{s}(t) = \langle e^t, e^t, e^t \rangle, 0 \leq t \leq 1$. Then $\mathbf{r}$ and $\mathbf{s}$ have

A. different velocities and different lengths

B. different speeds and the same length

C. different velocities and the same speed

D. different speeds and different lengths **E.** none of the above

(j) $\lim_{(x,y)\to(0,0)} \frac{xy}{x^2 + y^2}$

A. exists

B. exists along the lines $x = 0$ and $y = 0$, but not elsewhere

C. exists along the line $y = x$, but not elsewhere

D. does not exist **E.** none of the above

(k) The function $\mathbf{r}(t) = \left\langle \ln t, \frac{\sin t}{t}, \frac{1}{t} \right\rangle$

A. is continuous at $t = 0$ **B.** is continuous for all $t > 0$

C. is defined, but not continuous at $t = 0$

D. is discontinuous for all t **E.** none of the above

(l) The unit tangent vector of $\mathbf{r}(t) = \langle 1 + t^3, te^{-t}, \sin(2t)\rangle$ at $t = 0$ is

A. $(1/\sqrt{6})\langle 1, 1, 2\rangle$ **B.** $(1/\sqrt{5})\langle 0, 1, 2\rangle$ **C.** $(1/\sqrt{3})\langle 0, 1, 2\rangle$

D. $(1/\sqrt{14})\langle 3, 1, 2\rangle$ **E.** none of the above

(m) If $\mathbf{u}$ and $\mathbf{v}$ are vectors and the scalar projection of $\mathbf{v}$ onto $\mathbf{u}$ ($\text{comp}_{\mathbf{u}}\mathbf{v}$) is -2, then the angle between $\mathbf{u}$ and $\mathbf{v}$ is

A. acute **B.** $\pi/2$ **C.** obtuse **D.** π

E. none of the above

(n) The distance between the planes $x + y + z = 1$ and $x + y + z = 3$ is

A. 1 **B.** 2 **C.** $2/\sqrt{3}$ **D.** $3/\sqrt{3}$

E. none of the above

(o) If $\mathbf{u}$ is a unit vector that makes an angle of $\pi/4$ with $\mathbf{i}$ and $\pi/3$ with $\mathbf{k}$, then $\mathbf{u}$ can be

A. $\langle\frac{\sqrt{2}}{3}, \frac{2}{3}, \frac{\sqrt{3}}{3}\rangle$ **B.** $\langle\frac{2}{3}, \frac{\sqrt{3}}{3}, \frac{\sqrt{2}}{3}\rangle$ **C.** $\langle\frac{\sqrt{2}}{2}, \frac{1}{2}, -\frac{1}{2}\rangle$

D. $\langle\frac{\sqrt{2}}{2}, \frac{1}{2}, \frac{1}{2}\rangle$ **E.** none of the above