Math 81. *Abstract Algebra.*

Homework 3. Due on Wednesday, 1/27/2010.

1. Let F be a field of characteristic 0, and let m and n be distinct integers with $\sqrt{m} \notin F$, $\sqrt{n} \notin F$, and $\sqrt{mn} \notin F$.
   (a) Show that $[F(\sqrt{m}, \sqrt{n}) : F] = 4$.
   (b) Show by example that the above proposition is false if we only assume that $\sqrt{m} \notin F$ and $\sqrt{n} \notin F$.

2. Let $m_1, m_2, \ldots, m_t$ be distinct integers none of which are squares in $\mathbb{Z}$.
   (a) Show that $[\mathbb{Q}(\sqrt{m_1}, \sqrt{m_2}, \ldots, \sqrt{m_t}) : \mathbb{Q}] \leq 2^t$, and give an example to show that the inequality can be strict.
   (b) Now assume that the integers are square-free and relatively prime in pairs. Show that $[\mathbb{Q}(\sqrt{m_1}, \sqrt{m_2}, \ldots, \sqrt{m_t}) : \mathbb{Q}] = 2^t$.
   Hint: Use induction on t.

3. Consider the extension $\mathbb{Q}(\sqrt{2}, \sqrt{3}, \sqrt{5})/\mathbb{Q}$. Determine a basis for this extension.
   Hint: Rather than trying to prove directly that the set you write down is linearly independent, give an argument, based on what we have done in class, which proves your set is a basis.

4. Determine the degree of the extension $\mathbb{Q}(i, \sqrt{3}, e^{2\pi i/3})/\mathbb{Q}$, and write down three fields K with $\mathbb{Q} \subsetneq K \subsetneq \mathbb{Q}(i, \sqrt{3}, e^{2\pi i/3})$.

5. Problem 19, page 531 of the textbook.

6. Problem 20, page 531.

7. Problem 21, page 531.