# Math 63 Winter 2020 Takehome Midterm Exam

Instructor: Vladimir Chernov

Exam starts on Wednesday January 29.

The exam is due on Monday February 3 at class time.

PRINT NAME: ______________________________

The **Honor Principle** requires that you neither give nor receive any aid on this exam. You can use textbook by Rudin "Principles of mathematical analysis", your class notes, and solutions to the homework you did. You can **NOT** use other printed, electronic or similar sources to work on the exam. Use of calculators, computers, telephones and other electronic devices is not permitted. You must justify all of your answers to receive credit, so in particular please provide the reference to all the theorems from the textbook that you use. Please do all your work on the paper provided.

Grader's use only:

1. ________ /10

2. ________ /10

3. ________ /10

4. ________ /10

5. ________ /10

6. ________ /10

7. ________ /10

8. ________ /10

9. ________ /10

10. ________ /10

**Total:** ________ /100

1. Prove or disprove the following statement: $\sqrt{20}$ is not a rational number.

2. Let $A$ be a nonempty set of real numbers $A \subset \mathbb{R}$ which is bounded above and let $r > 0$ a positive real number. We define the set $rA = \{rx | x \in A\}$. Is it always true that $\sup(rA) = r \sup A$? Prove your answer.

3. We call a map $f : X \to Y$ from one metric space to another **nice** if for every open $U \subset Y$ we have that $f^{-1}(U)$ is open in $X$. Show that an image of a compact set $E \subset X$ under a nice map is compact.

4. Let $f : X \to Y$ be a nice map from a compact metric space $X$ to a metric space $Y$ which is a bijection. Is it true that the inverse map $f^{-1} : Y \to X$ is always nice. Recall that a map $f : X \to Y$ from one metric space to another **nice** if for every open $U \subset Y$ the set $f^{-1}(U)$ is open in $X$. You may want to use the result of the previous problem.

5. Let $A$ be the set of all roots of finite degree polynomials whose coefficients are square roots of rational numbers. Is the set $A$ countable? Prove your answer.

6. Let $A_1, A_2, A_3, \ldots$ be subsets of a metric space. If $B_n = \cup_{i=1}^{n} A_i$ then prove that $\bar{B}_n = \cup_{i=1}^{n} \bar{A}_i$ for $n = 1, 2, 3, \ldots$. Give an example where $\overline{\cup_{i=1}^{\infty} A_i}$ is strictly larger than $\cup_{i=1}^{\infty} \bar{A}_i$.

7. For a susbet $E$ in a metric space we denote by $E^\circ$ the *interior* of $E$, which is defined as the union of all open susbets of $E$. Is it possible to have the situation where $(\bar{E})^\circ$ is strictly larger than $E^\circ$? Prove your answer.

8. For $x, y \in \mathbb{R}$ we define $d(x, y) = (x - y)^2$. Is this a metric or not? Prove your asnwer.

9. Let $A$ be a star convex susbet in $\mathbb{R}^n$. Prove that it has to be connected or find a counterexample.

10. Let $f : X \to Y$ be a nice map from a metric space to a metric space. Prove or disprove the following statement: an image of a nonempty connected set under a nice map is always connected. Recall that a map $f : X \to Y$ is **nice** if for every open $U \subset Y$ we have that $f^{-1}(U)$ is open in $X$.