Group Work Tip #1. Before moving on to the next problem, make sure that everybody understood the solution. One member of the group should try to summarize the method(s) employed.

8. Topology of $\mathbb{R}^n$

Exercise 8.1. Show that the open sets of $\mathbb{R}^n$ (as defined in class or as in Definition 4.3.6) satisfy the axioms ($\tau 1$), ($\tau 2$), and ($\tau 3$) of a topology.

Exercise 8.2. Give examples of open and closed subsets of $\mathbb{R}$, $\mathbb{R}^2$, and $\mathbb{R}^n$.

Exercise 8.3.

> **Definition.** A point $\mathbf{x}$ is a **limit point** of $A \subseteq \mathbb{R}^n$ if there is a sequence $(\mathbf{x}_k)_{k=1}^{\infty}$, with $\mathbf{x}_k \in A$, such that $\mathbf{x} = \lim_{k\to\infty} \mathbf{x}_k$.

Show that a subset A of $\mathbb{R}^n$ is **closed** (that is, its complement is an open subset of $\mathbb{R}^n$) if and only if A contains all its limit points.

Exercise 8.4. Let X be a complete metric space. Show that $A \subseteq X$ is complete iff A is a closed subset.

Note. Read carefully Section 4.3 in the text-book. Read carefully Example 4.3.10. There are subsets of $\mathbb{R}^n$, actually most of them, which are neither open nor closed!