Exercise 14.4. For every bounded function $f : [a, b] \to \mathbb{R}$, let

$$L(f) = \sup_P L(f, P) \quad \text{and} \quad U(f) = \inf_P U(f, P).$$

Show that $L(f) \leq U(f)$.

Exercise 14.5. Prove:

Riemann's condition for integrability. Let $f : [a, b] \to \mathbb{R}$ be a bounded function. The following are equivalent:

(a) f is Riemann integrable;
(b) for every $\varepsilon > 0$, there exists a partition P such that $U(f, P) - L(f, P) < \varepsilon$.

Exercise 14.6. Every monotone function on $[a, b]$ is Riemann integrable.