Group Work Tip #2. Encourage everybody in the group to participate in the discussion and listen to what the others are saying.

Exercise 13.3. Prove:

> **Rolle's Theorem.** *Suppose that f is a function that is continuous on $[a, b]$ and differentiable on (a, b), such that $f(a) = f(b) = 0$. Then there is a point $c \in (a, b)$ such that $f'(c) = 0$.*

Assume next that $f(a) \neq f(b)$. What would be the equivalent of Rolle's Theorem in this case?

Exercise 13.4. Prove:

> **Mean Value Theorem.** *Suppose that f is a function that is continuous on $[a, b]$ and differentiable on (a, b). Then there is a point $c \in (a, b)$ such that*
> $$f'(c) = \frac{f(b) - f(a)}{b - a}.$$

Exercise 13.5. Let f be a differentiable function on $[a, b]$.

(a) If $f'(x) > 0$, for all $x \in (a, b)$, then f is strictly increasing.
(b) If $f'(x) = 0$, for all $x \in (a, b)$, then f is constant.

Exercise 13.6. Let f be a differentiable function on $[a, b]$, with $f'(a) < 0 < f'(b)$.

(a) Show that there are points c and d, with $a < c < d < b$, such that $f(c) < f(a)$ and $f(d) < f(b)$.
(b) Show that the minimum of f on $[a, b]$ occurs at an interior point.
(c) Conclude that f' takes closed intervals onto closed intervals. This is known as the **property of Darboux**. Note that f' is *not* assumed to be continuous but simply to exist everywhere on $[a, b]$.

Exercise 13.7. (**Fun Problem.**) Let $f, g : [a, b] \to \mathbb{R}$ be two continuous functions, differentiable on (a, b). Assume that g and g' are nowhere zero on (a, b) and that $f(a)/g(a) = f(b)/g(b)$. Prove that there exists $c \in (a, b)$ such that

$$\frac{f(c)}{g(c)} = \frac{f'(c)}{g'(c)}.$$