Exercise 12.7. Consider metric spaces X and Y, and a continuous function $f : X \to Y$. Show that for every compact subset C of X, its image under f, $f(C)$, is compact in Y.

Exercise 12.8. Consider metric spaces X and Y, and a continuous function $f : X \to Y$. If C is compact in Y, must $f^{-1}(C)$ be compact?

Exercise 12.9. Consider yet another definition of compactness:

> **Definition.** Let X be a topological space and Y a subset of X. A collection of open subsets $\{U_\alpha \,|\, \alpha \in A\}$ in X is called an **open cover of** Y if $Y \subset \bigcup_{\alpha \in A} U_\alpha$. A **subcover** of $\{U_\alpha \,|\, \alpha \in A\}$ is just a subcollection $\{U_\alpha \,|\, \alpha \in B\}$, with $B \subset A$, that is still a cover.
>
> **Definition.** A subset C of X is called **compact** if
>
> (K3) every open cover of C has a *finite* subcover.

Prove the property of Exercise 12.7 using this new description of compactness.