Exercise 12.3. Let (X, ρ_X) and (Y, ρ_Y) be two metric spaces. For a function $f : X \to Y$ show that the following are equivalent:

(a) f is continuous (on X);

(b) for every convergent sequence $(x_n)_n$ in X, with $\lim_{n\to\infty} x_n = x$, it follows that

$$\lim_{n\to\infty} f(x_n) = f(x).$$

Exercise 12.4. Recall the definition of an open set in a metric space. Let (X, ρ_X) and (Y, ρ_Y) be two metric spaces. If $f : X \to Y$ is continuous, and $U \subset Y$ is open, then what can you say about the subset $f^{-1}(U)$ of X? (*Hint.* Use the ε-δ-definition of continuity.)

Exercise 12.5.

(a) Show that the sum of two continuous functions with the same domain is a continuous function.

(b) Show that the product of two scalar valued functions is continuous.

(c) Show that the composition of two continuous functions is a continuous function.