## 1. Functions

**Exercise 1.1.** Consider two non empty sets $A$ and $B$.

(a) Show that one can define a **function** $f$ **from** $A$ **to** $B$ as a subset of $A \times B$, denoted $G(f)$, such that for each $a \in A$ there is *exactly one* $b \in B$ such that $(a, b) \in G(f) \subset A \times B$.
(b) Interpret the definition by using a picture. What would be an appropriate name for $G(f)$?

**Exercise 1.2.** Consider a bijective function $f : A \to B$. Using Exercise 1.1 construct the inverse $f^{-1}$ of $f$, and interpret this construction in terms of cartesian products of sets.

**Exercise 1.3.** By definition, the **power set** $P(X)$ of a set $X$ is the set consisting of all subsets of $X$, including $\emptyset$.

(a) Find $P(X)$ for $X = \{1\}$, $X = \{1, 2\}$, $X = \{1, 2, 3\}$, and $X = \{1, 2, 3, \ldots, n\}$.
(b) How many different subsets of $X = \{1, 2, 3, \ldots, n\}$ are there?
(c) Find a bijection between $P(X)$ and the set of all functions $f : X \to \{0, 1\}$. Can you recover the answer from part (b)?